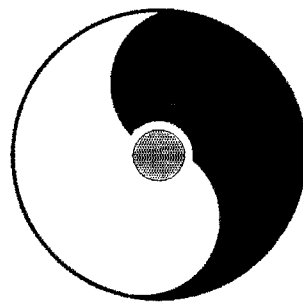


Hadron Structure from Lattice QCD

March 18 - 22, 2002



Organizers:

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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

During the first year, the Center had only a Theory Group. In the second year, an Experimental Group was also established at the Center. At present, there are seven Fellows and eight Research Associates in these two groups. During the third year, we started a new Tenure Track Strong Interaction Theory RHIC Physics Fellow Program, with six positions in the first academic year, 1999-2000. This program has increased to include ten theorists and one experimentalist in the current academic year, 2001-2002. Beginning this year there is a new RIKEN Spin Program at RBRC with four Researchers and three Research Associates.

In addition, the Center has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form proceedings, which can therefore be available within a short time. To date there are forty-two proceeding volumes available.

The construction of a 0.6 teraflops parallel processor, dedicated to lattice QCD, begun at the Center on February 19, 1998, was completed on August 28, 1998.

T. D. Lee
August 2, 2001

*Work performed under the auspices of U.S.D.O.E. Contract No. DE-AC02-98CH10886.

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Hadron Structure from Lattice QCD

Introduction

The RIKEN BNL Research Center workshop on “Hadron Structure from Lattice QCD” was held at BNL during March 11-15, 2002. Hadron structure has been the subject of many theoretical and experimental investigations, with significant success in understanding the building blocks of matter. The nonperturbative nature of QCD, however, has always been an obstacle to deepening our understanding of hadronic physics. Lattice QCD provides the tool to overcome these difficulties and hence a link can be established between the fundamental theory of QCD and hadron phenomenology. Due to the steady progress in improving lattice calculations over the years, comparison with experimentally measured hadronic quantities has become important. In this respect the workshop was especially timely. By providing an opportunity for experts from the lattice and hadron structure communities to present their latest results, the workshop enhanced the exchange of knowledge and ideas. With a total of 32 registered participants and 26 talks, the interest of a growing community is clearly exemplified.

At the workshop Schierholz and Negele presented the current status of lattice computations of hadron structure. Substantial progress has been made during recent years now that the quenched results are well under control and the first dynamical results have appeared. In both the dynamical and the quenched simulations the lattice results, extrapolated to lighter quark masses, seem to disagree with experiment. Melnitchouk presented a possible explanation (chiral logs) for this disagreement. It became clear from these discussions that lattice computations at significantly lighter quark masses need to be performed.

Recently, much work has been done using chiral perturbation theory (quenched, and partially quenched) in an attempt to connect the lattice calculations of hadronic matrix elements with the physically interesting light quark regime. Ji, Chen, and Savage gave very instructive talks on their latest results regarding this topic.

Apart from talks on structure functions and parton distribution functions (see also the talk by Sasaki), which mostly pertain to the structure of the lightest baryons, a review of the current status of the excited baryon spectrum on the lattice was given by Richards. Michael presented results on mesons, in particular flavor singlet and exotic mesons.

On the first day, Brodsky and Sterman reviewed theoretical developments in hadronic physics. Their talks were particularly stimulating by drawing attention to some limitations of current lattice techniques (see also the talk by Boer).

Furthermore, there were talks on topological features of lattice QCD (Liu, Blum) and of hadrons (Faccioli). Some more technical lattice talks were presented, especially regarding new developments in the use of domain wall fermions (Orginos, Izubuchi), the Schrödinger functional scheme (Palombi, Aoki) and nonperturbative renormalization (Dawson, Aoki, Rakow). Brower presented the modern picture of CFT/AdS duality and compared the AdS glueball spectrum for strong coupling QCD_4 with the existing lattice data.

Finally, Ogawa and Goto presented experimental progress concerning the measurements

of gluon polarization and quark transversity at RHIC and KEK, which will allow for comparisons to existing and future lattice evaluations of those quantities.

All in all, the workshop has been a great success. The organizers are grateful to all the participants who came to the Center, and who contributed to the success of the workshop with their interesting talks, which stimulated lively discussions. We would also like to thank Prof. T.D. Lee and the RIKEN-BNL Research center for their encouragement and generous support. Last but not least, sincere thanks to Pamela Esposito for her invaluable help in organizing and running this pleasant workshop.

Tom Blum
Daniel Boer
Mike Creutz
Shigemi Ohta
Kostas Orginos

Hadron Structure from QCDSF

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RIKEN-BNL Workshop

March 18 - 22, 2002

Summary

The lattice formulation of QCD is at present the only known way of obtaining low-energy properties of the theory without any model assumptions. My group, the QCDSF collaboration, has been actively involved for the last few years in determining moments of the polarized and unpolarized structure functions of the nucleon.

The moments of the polarized structure functions turn out to be in good agreement with experiment. Let me give two examples: On p. 6 I show the axial vector coupling of the nucleon g_A , i.e. the lowest moment of g_1 , and on p. 7 I show d_2 , the twist-3 contribution to the moment $\int_0^1 dx x^2 g_2(x, Q^2)$.

To compare the lattice results with the experimental numbers, one must extrapolate the data from the lowest calculated quark mass to the physical value. While a simple, linear extrapolation seems to work well in the polarized case, it overestimates the experimental number by 40% in case of $\langle x \rangle$, suggesting that important physics is being omitted. On p. 3 I show the lattice data for quenched QCD, and on p. 4 for the full theory. There is not much difference between the two cases.

The chiral effective theory (p. 3) suggests that the nucleon's pion cloud gives rise to non-analytic terms which may result in a large deviation from linearity at quark masses much smaller than probed up to now in lattice calculations. Preliminary results of a recent study in the quenched approximation on large lattices and very small pion masses (about twice as large as the physical pion mass) indicate (p. 5) that this might indeed be the case.

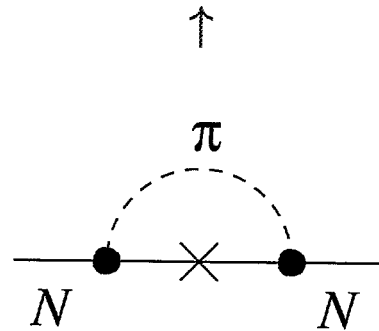
$$\langle x^n \rangle_{NS} = A \left(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right) \right) + \text{analytic terms}$$

$$\Delta^n q_{NS} = A \left(1 - \frac{2g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right) \right) + \text{analytic terms}$$

Thomas, Melnitchouk & Steffens

Arndt & Savage

Chen & Ji



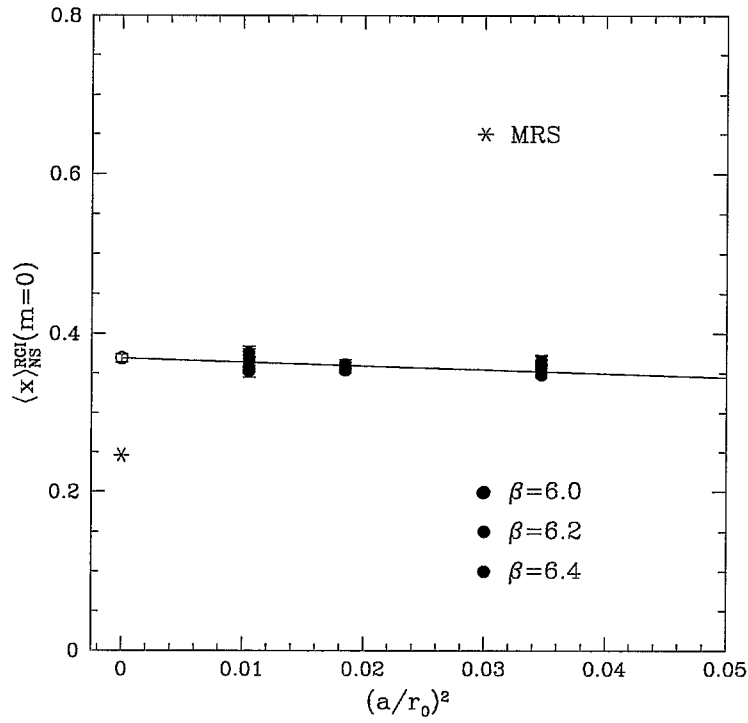
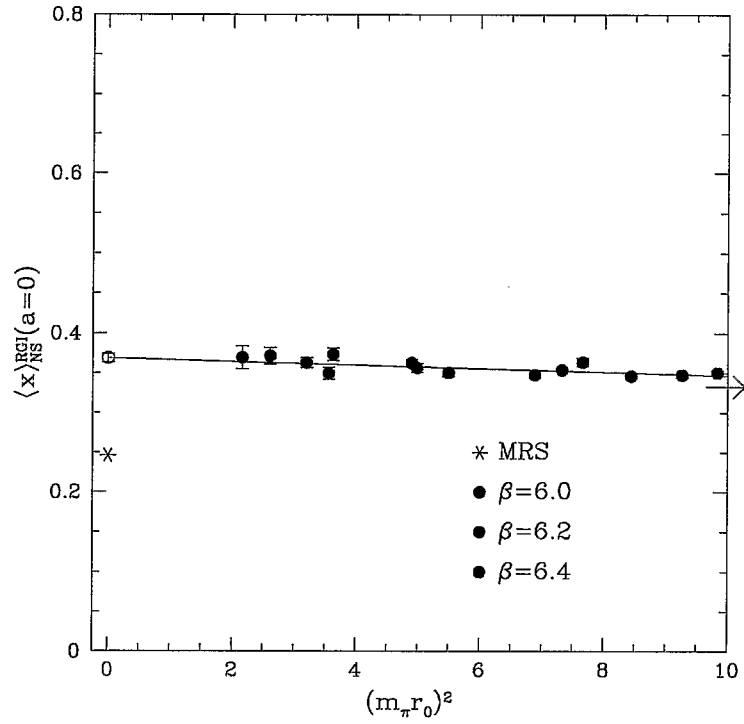
Quenched

$$\langle x^n \rangle_{NS} = A \left(1 - 0.28 r_0^2 m_\pi^2 \ln \left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right) \right) + \text{analytic terms}$$

$$r_0 = 0.5 \text{ fm}$$

Chen & Savage

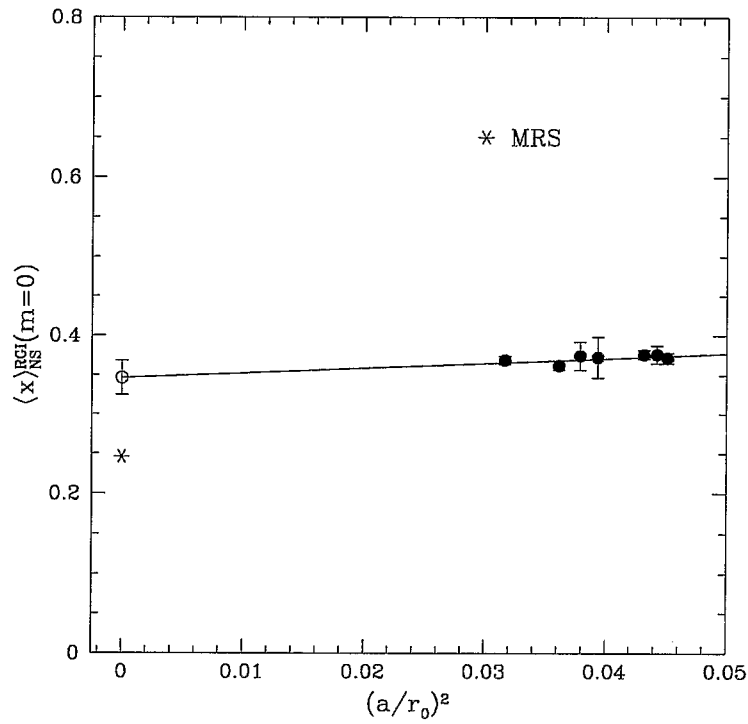
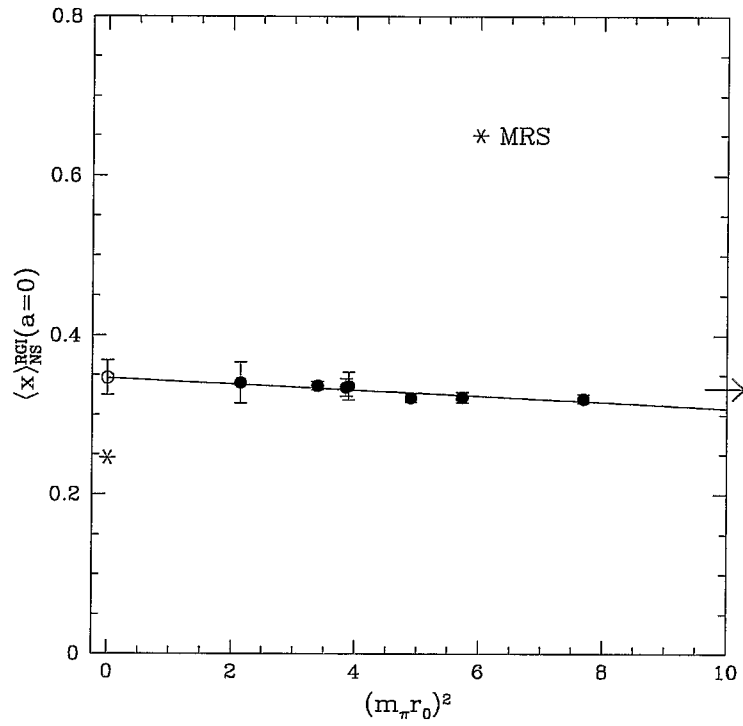
$\langle x \rangle$ Quenched NS, RGI



Fit ansatz: $\langle x \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$,
analytic

$r_0 = 0.5$ fm

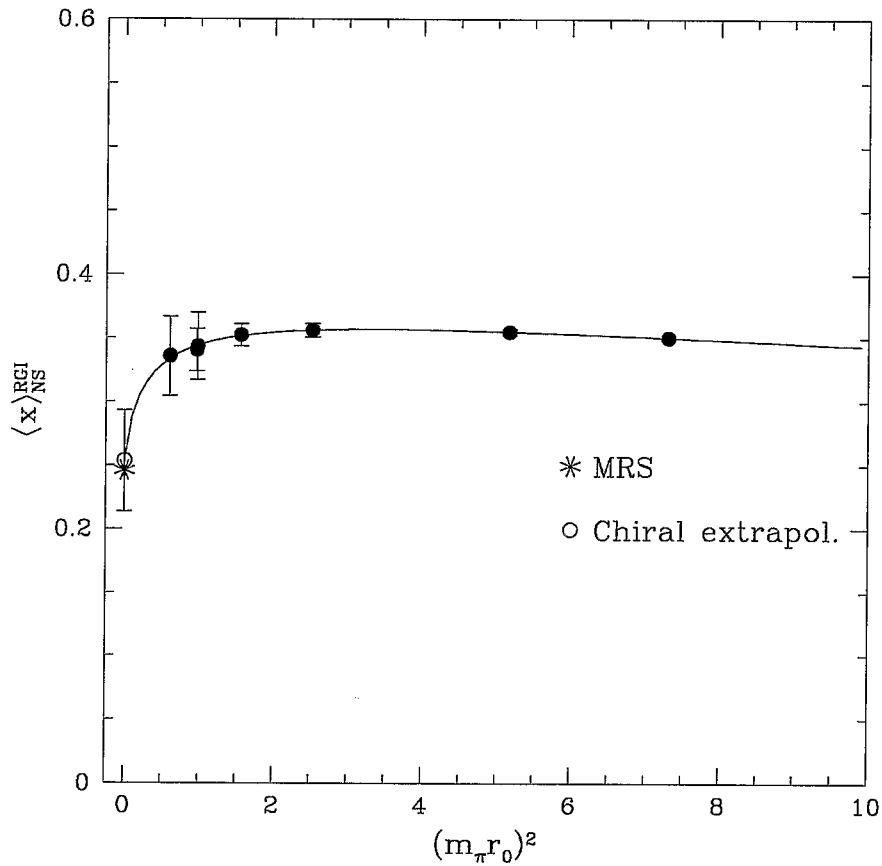
$\langle x \rangle \quad N_f = 2 \quad \text{NS, RGI}$



Fit ansatz: $\langle x \rangle = A + B(m_\pi r_0)^2 + C(a/r_0)^2$

True chiral behavior ?

Quenched, $\beta = 6.0$, small m



A fit to

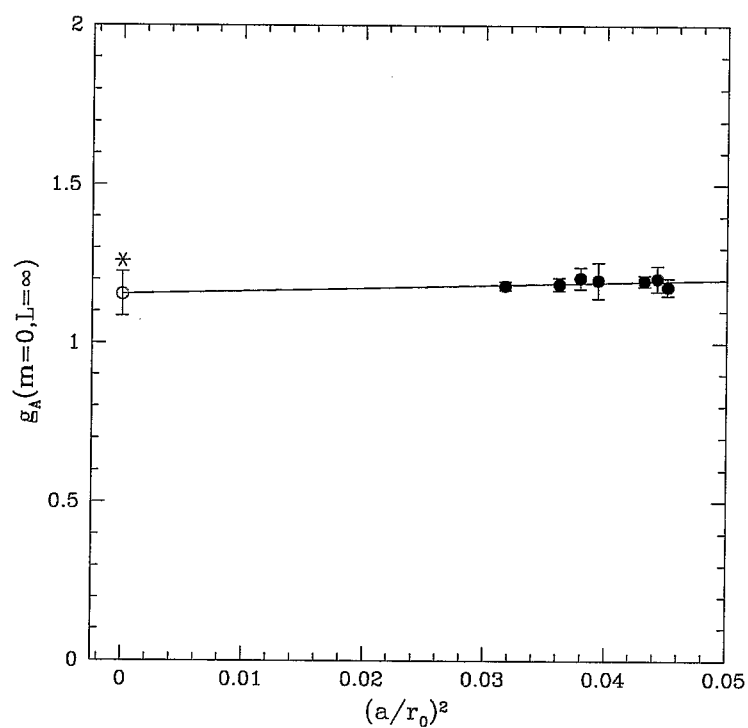
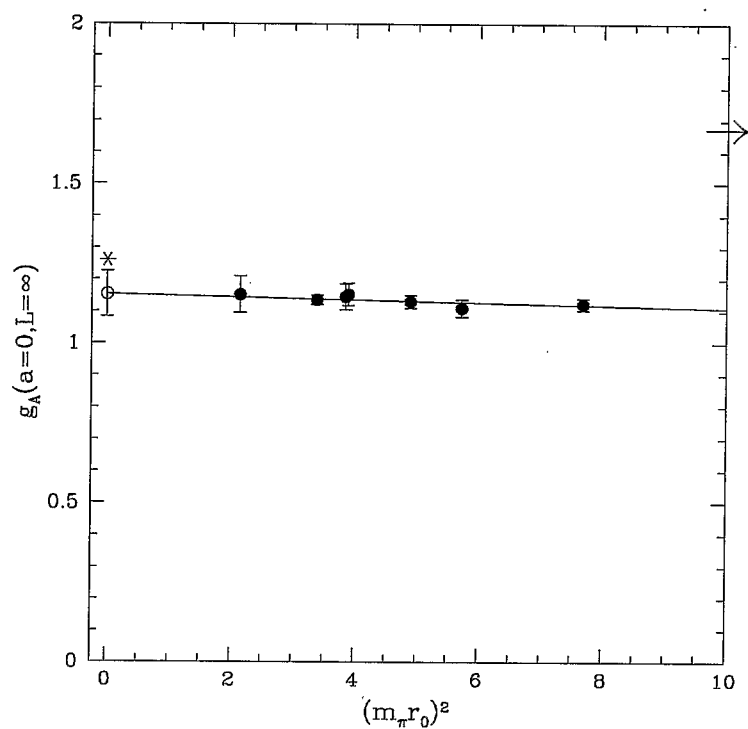
$$\langle x \rangle = A \left(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \left(\frac{m_\pi^2}{m_\pi^2 + \Lambda^2} \right) \right) + B(m_\pi r_0)^2$$

including the (wrong) non-analytic term is now possible and gives $\Lambda \approx 350$ MeV. ✓

Lesson: To make contact with chiral perturbation theory, one will in general have to do simulations at dynamical quark (pion) masses of $m \approx 20$ MeV ($m_\pi \approx 300$ MeV), so that the parameters of the non-linear expansion are well determined by the lattice calculation.

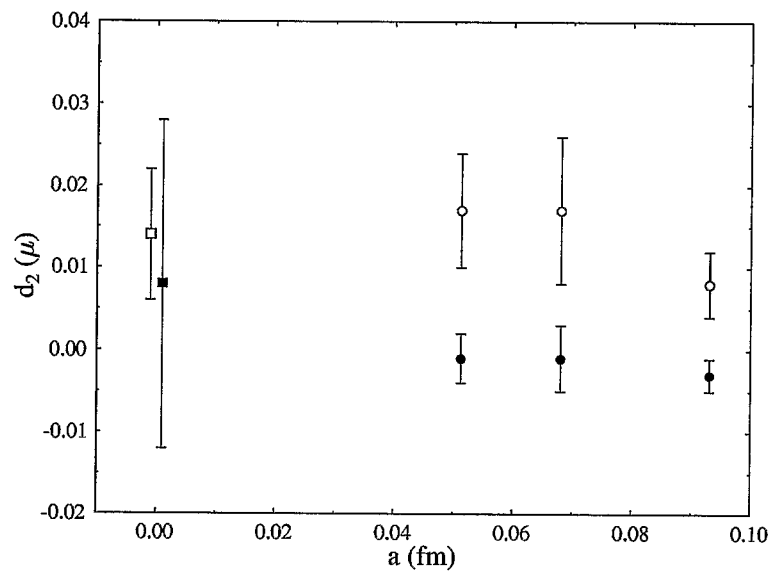
$$g_A \quad N_f = 2$$

Extrapolated to infinite volume



Λ is not a universal parameter, but may depend on the process !
Higher orders will have to tell.

d_2 Quenched



↑
Experiment

○ proton

● neutron

$\mu^2 = 5 \text{ GeV}^2$

$Q^2 \gtrsim 5 \text{ GeV}^2$:

$$\int_0^1 dx x^2 g_2(x, Q^2) \approx -\frac{3}{2} \int_0^1 dx x^2 g_1(x, Q^2)$$

Wandzura-Wilczek

Expect deviations at smaller Q^2 .

Light-Front Methods and Non-Perturbative QCD *

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Invited talk presented at the
Workshop on Hadron Structure from Lattice QCD
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Brookhaven National Laboratory, Upton, New York
March 18-22, 2002

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1 Introduction

In this talk I will briefly discuss some features of light-front quantization methods for nonperturbative QCD in comparison with traditional lattice methods. Some of the novel features and new directions are illustrated in the accompanying transparencies.

A central focus of non-perturbative light-front methods in QCD is the set of light-front Fock state wavefunctions $\psi_n^H(x_i, k_{i\perp}, \lambda_i)$, which represent a hadron in terms of its quark and gluon degrees of freedom [1]. Here $x_i = \frac{k_i^+}{P^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$ is the boost-invariant light-cone momentum fraction with $\sum_{i=1}^n x_i = 1$. The λ_i represent values of the spin-projections S_i^z of the constituents. In principle, the light-front wavefunctions of hadrons can be computed by diagonalizing the QCD Hamiltonian H_{LF} quantized at fixed $\tau = t + z/c$ in light-cone gauge $A^+ = 0$ [2]. The set of ψ_n^H wavefunctions are then obtained as the projections of the hadron's eigenstate on the n -particle Fock states. Remarkably, the light-front wavefunctions are frame independent; *i.e.*, independent of the hadron's total momentum P^+ and P_\perp . Given these amplitudes, one can calculate many hadronic processes from first principles. The sum of squares of the light-front wavefunctions give the quark and gluon distributions $q_H(x, Q)$ and $g_H(x, Q)$ at resolution Q , including all spin measures and correlations, as well as the unintegrated distributions $q_H(x, k_\perp)$ and $g_H(x, k_\perp)$ controlling inclusive reactions. Form factors and exclusive weak decay matrix elements have exact representations. Similarly, the deeply virtual Compton amplitude $\gamma p \rightarrow \gamma p'$ can be expressed as overlap integrals $n = n', n = n' + 2$ of the initial and final light-front proton wavefunctions [3]. The hadron distribution amplitudes $\phi^H(x_i, Q)$ which control hard exclusive processes, including exclusive B decays, are the transverse momentum integrals of the lowest particle number valence Fock state wavefunction [4].

The light-front quantization of QCD can be carried out with rigor using the Dirac method to impose the light-cone gauge constraint and eliminate dependent degrees of freedom [5]. Unlike the case in equal time quantization, the vacuum remains trivial. One can verify the QCD Ward identities for the physical light-cone gauge and compute the QCD β function. Recently, Srivastava and I [6] have extended the light-front quantization procedure to the Standard Model. The spontaneous symmetry breaking of the gauge symmetry is due to a zero mode of the scalar field rather than vacuum breaking. The Goldstone component of the scalar field provides mass to the W^\pm and Z^0 gauge bosons as well as completing its longitudinal polarization. The resulting theory is free of Faddeev-Popov ghosts and is unitary and renormalizable.

The light-front Hamiltonian has been diagonalized for a number of $1+1$ theories including QCD(1+1) [7] and supersymmetric theories [8] using the discretized light-cone quantization (DLCQ) method, which discretizes the momentum variables $k_i^+ = \frac{2\pi}{L}n_i$, $P^+ = \frac{2\pi}{L}K$ with $\sum n_i = K$ truncates the Fock space, while retaining the essential Lorentz symmetries of the theory. The continuum limit is approached as the harmonic resolution $K \rightarrow \infty$. Model $3+1$ theories are also being solved using DLCQ and Pauli-Villars fields as regulators [9]. A new Pauli-Villars regularization method for QCD

has been developed by Franke *et al.* [10].

Thus light-front quantization of QCD can provide a rigorous alternative to lattice methods. It provides nonperturbative solutions to the hadronic bound state and continuum solutions to the spectrum and the corresponding wavefunctions in Minkowski space. There are no fermion-doubling problems or finite-size effects. However, the diagonalization of the LF Hamiltonian is computationally challenging for QCD, an area which could greatly benefit from the expertise and computer technology of the lattice community. A possible alternative is to use variational methods to minimize the expectation value of the light-front Hamiltonian. The trial wavefunction can be constrained by noting that the numerator structure of the individual Fock state wavefunctions are largely determined by the relative orbital angular momentum and J^z conservation. Ladder relations relate Fock states differing by one or two gluon quanta [11]. Other alternative methods have been developed including the transverse lattice which combines DLCQ(1+1) with a lattice in transverse space [12, 13, 14].

The light-front partition function, summed over exponentially-weighted light-front energies, has simple boost properties which may be useful for studies in heavy ion collisions [15].

One of the important tests of Lorentz invariance is the vanishing of the anomalous gravitomagnetic moment $B(0)$ for any spin-half system; *i.e.*, the ratio of the spin precession frequency of a particle to its Larmor frequency is exactly 2 if the magnetic field is replaced by a gravitational field. This is a consequence of the equivalence theorem of general relativity [16]. The $B(q^2)$ form factor is defined from the spin-flip matrix element of the energy momentum tensor. Hwang, Ma, Schmidt and I have shown that $\sum_i^n B_i(0) = 0$, Fock state by Fock state, in the light-front representation [17]. It is an important challenge to lattice gauge theory to verify $B(0) = 0$ for a proton, since any nonzero result will provide a diagnostic of finite lattice size and other systematic errors.

The light-front method also suggests the possibility of developing an “event amplitude generator” by calculating amplitudes for specific parton spins using light-front time-ordered perturbation theory [18]. The positivity of the k^+ light-front momenta greatly constrains the number of contributing light-front time orderings. The renormalized amplitude can be obtained diagram by diagram by using the “alternating denominator” method [19] which automatically subtracts the relevant counterterm. The resulting amplitude can be convoluted with the light-front wavefunctions to simulate hadronization and hadron matrix elements.

Recently, Hoyer, Marchal, Peigne, and Sannino and I [20] have challenged the common view that structure functions measured in deep inelastic lepton scattering are determined by the probability of finding quarks and gluons in the target. We show that this is not correct in gauge theory. Gluon exchange between the fast, outgoing partons and target spectators, which is usually assumed to be an irrelevant gauge artifact, affects the leading twist structure functions in a profound way. This observation removes the apparent contradiction between the projectile (eikonal)

and target (parton model) views of diffractive and small x_{Bjorken} phenomena. The diffractive scattering of the fast outgoing quarks on spectators in the target in turn causes shadowing in the DIS cross section. Thus the depletion of the nuclear structure functions is not intrinsic to the wave function of the nucleus, but is a coherent effect arising from the destructive interference of diffractive channels induced by final state interactions. This is consistent with the Glauber-Gribov interpretation of shadowing as a rescattering effect.

It is an interesting question whether the moments of structure functions obtained from lattice gauge theory can account for the nuclear shadowing phenomenon, considering that shadowing depends in detail on the phase structure of diffractive and deep inelastic scattering amplitudes.

Recent measurements from the HERMES and SMC collaborations show a remarkably large azimuthal single-spin asymmetries A_{UL} and A_{UT} of the proton in semi-inclusive pion leptonproduction $\gamma^*(q)p \rightarrow \pi X$. Recently, Dae Sung Hwang and Ivan Schmidt and I [21] have shown that final-state interactions from gluon exchange between the outgoing quark and the target spectator system lead to single-spin asymmetries in deep inelastic lepton-proton scattering at leading twist in perturbative QCD; *i.e.*, the rescattering corrections are not power-law suppressed at large photon virtuality Q^2 at fixed x_{bj} . The existence of such single-spin asymmetries requires a phase difference between two amplitudes coupling the proton target with $J_p^z = \pm \frac{1}{2}$ to the same final-state, the same amplitudes which are necessary to produce a nonzero proton anomalous magnetic moment. We show that the exchange of gauge particles between the outgoing quark and the proton spectators produces a Coulomb-like complex phase which depends on the angular momentum L^z of the proton's constituents and is thus distinct for different proton spin amplitudes. The single-spin asymmetry which arises from such final-state interactions does not factorize into a product of distribution function and fragmentation function, and it is not related to the transversity distribution $\delta q(x, Q)$ which correlates transversely polarized quarks with the spin of the transversely polarized target nucleon. These effects highlight the unexpected importance of final and initial state interactions in QCD observables.

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Light-Front Methods and
Non-Perturbative QCD

Stan Brodsky

SLAC

Hadron Structure from Lattice QCD.

BNL - Riken

March 18, 2002

Light-Cone Wavefunctions

encode all helicity, transversity
distributions

$$Q_{\lambda/\lambda_P} = \left| \int \left[\begin{array}{c} \text{diagram: a circle with an incoming arrow from the left labeled } \lambda_P, \text{ and several outgoing arrows to the right. One arrow is labeled } x, \lambda. \end{array} \right] \right|^2$$

$$Q_{\lambda/\lambda_P}(x, \Lambda) \quad \left\{ \begin{array}{l} \text{transversity: density} \\ \text{motion} \\ \text{light-cone helicity} \end{array} \right.$$

$$= \sum_{n, \xi} \int \left| \psi_{n, \lambda_P}^{(\Lambda)}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{i=1} \bar{\pi} dx_i \prod_{j=1} \bar{\pi} d^2 k_{\perp j}$$

$$\delta(\sum x_i - 1) \delta(\sum \vec{k}_{\perp i})$$

$$\delta(x - x_q) \delta_{\lambda, \lambda_q}$$

$$\Theta(\Lambda^2 - m_n^2)$$

Feynman
Light-cone Scheme

$$\tau = t + z/c$$

Dirac
Bjorken, Kogut, Soper
LePage + SSB
Pauli + SSB

Equation of motion

$$i \frac{\partial}{\partial \tau} |\Psi_H\rangle = P^- |\Psi_H\rangle = \frac{M_H^2 + P_\perp^2}{P^+} |\Psi_H\rangle$$

$$H_{LC} = P^- P^+ - P_\perp^2$$

P^+, P_\perp
kinematical

$$H_{LC} |\Psi_H\rangle = M_H^2 |\Psi_H\rangle$$

⇒ eigenvalue problem for LC Hamiltonian

Insert complete set of H_{LC}^0 eigenstates ^{color-singlet}

$$\sum_n |n\rangle \langle n| = \mathbb{I}$$

$$\sum_n \langle m | H_{LC} | n \rangle \langle n | \Psi_H \rangle = M_H^2 \langle m | \Psi_H \rangle$$

DLCQ

⇒ Heisenberg matrix form of eigenvalue problem

$$|\Psi_H\rangle = \sum_n |n\rangle \langle n | \Psi_H \rangle = \sum_n |n\rangle \psi_{n/H}(x_i, \vec{k}_{i\perp}, \lambda_i)$$

⇒ LC Fock expansion of eigenstate $|\Psi_H\rangle$

* Given $\{\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$

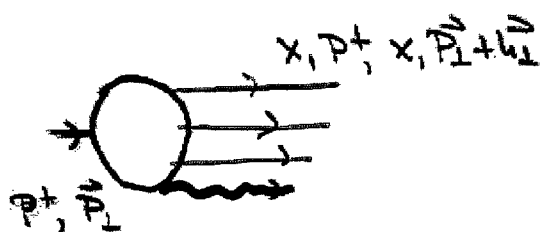
wavefunction known for all P^μ !

relative coordinates

$$|P^+, \vec{P}_\perp\rangle = \sum_n \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \prod_j \frac{1}{\sqrt{x_j}}$$

$$|x_i, P^+, x_i, \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

absolute coordinates



In equal-time theory (instant form)

boosts mix with interactions

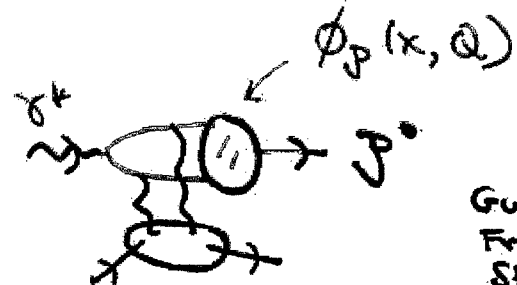
changing $\vec{P} \rightarrow \vec{P}'$ is complicated

as solving $H|\Psi\rangle = E|\Psi\rangle$

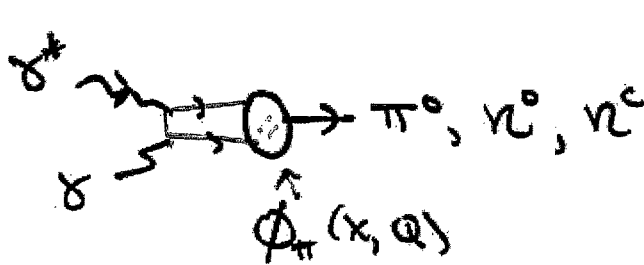
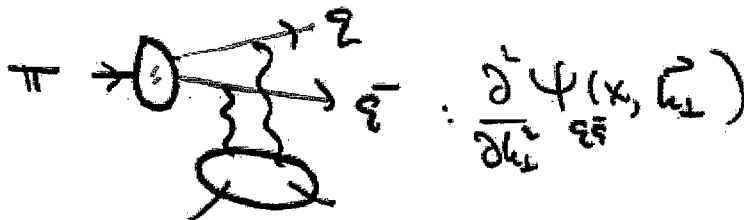
L.e. wfs - $\left\{ \begin{array}{l} \text{rest frame} \\ P^+ \neq 0 \end{array} \right.$
Frame-independent!

Recent Applications of Ψ_{LC}

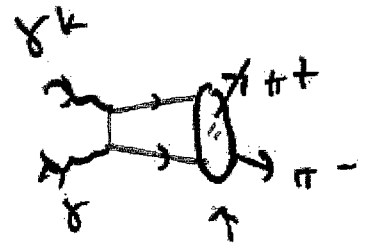
Diffractive Processes



Gunion
Frankfurt
Strohmaier
Muller
SJB
Collins



Lepreux, SJB



Dubl, et al

Deeply virtual Compton scatt

Einhorn
Burkhardt
QCD (H1) $N_c \rightarrow \infty$



\Rightarrow



X. Ji
A. Radyushkin
Close, Gunion, SJB

Huang
Schnitz
Ma
SJB

- * Non-forward parton distributions and Ψ_{LC}
- * Sum rules, * Jt cons.
- * Relations to EM, Gravity sum rules
- * Tests on quantum fluctuations of electron

QCD (3+1)

| Sector | Class | 0 | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
|--------|-------|---|---|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | 0 | 0 | | | | | | | | | | | | | | | | |
| 2 | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| 3 | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| 4 | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| 5 | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| 6 | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |
| | g | | g | q ₁ | q ₂ | q ₃ | q ₄ | q ₅ | q ₆ | q ₇ | q ₈ | q ₉ | q ₁₀ | q ₁₁ | q ₁₂ | q ₁₃ | q ₁₄ | q ₁₅ |

DLQ: $\langle n | H_L^H | m \rangle$

$$(m^2 - \sum_n \frac{k_n^2 + m^2}{x}) \psi_n = \sum_m \langle n | H_L^H | m \rangle \psi_m$$

HCP
+

PBC: $k^+ = \frac{2\pi}{L} n$, $k_\perp = \frac{2\pi}{L_\perp} n_\perp$

H.C. Pauli
8JB

DLCQ (3+1)

Periodic boundary conditions

$$-L < x^- < L, \quad -L_\perp < x, y < L_\perp$$

\Rightarrow Discrete momenta:

$$P^- = \frac{\pi}{L} n_i, \quad \vec{P}_\perp = \frac{\pi}{L_\perp} (n_x, n_y)$$

$$P^+ = \frac{\pi}{L} k, \quad \sum_i n_i = k$$

$$x_i = \frac{P^-}{P^+} = \frac{n_i}{k} > 0$$

* Each state number limited by k

$$\sum_i \frac{m_i^2 + P_{\perp i}^2}{x_i} < \Lambda^2 \quad \text{limit } n_x, n_y$$

* Continuous limit $k \rightarrow \infty$

Applications to Yukawa theory (3+1)

Hille, McCartin, 8JB

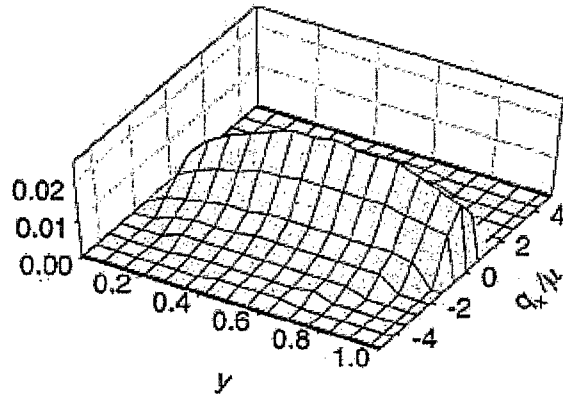


FIG. 13. The parallel-helicity, one-boson amplitude $\phi_{\sigma\sigma}^{(1,0,0)}$ as a function of longitudinal momentum fraction y and one transverse momentum component q_x in the $q_y = 0$ plane. The parameter values are $K = 29$, $N_1 = 7$, $M = \mu$, $\Lambda^2 = 50\mu^2$, $\mu_1^2 = 10\mu^2$, $\mu_2^2 = 20\mu^2$, $\mu_3^2 = 30\mu^2$, and $\langle\phi^2(0)\rangle = 0.25$.

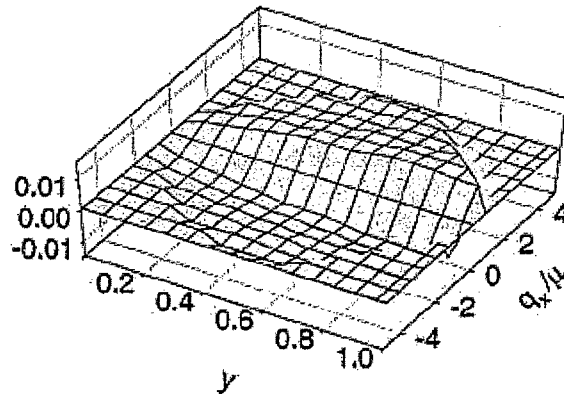


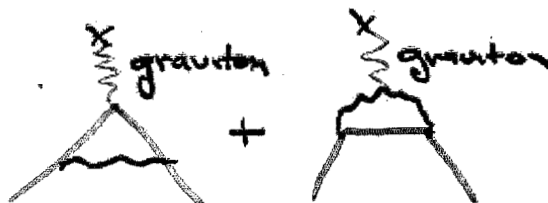
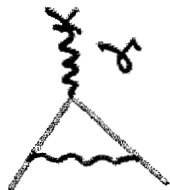
FIG. 14. Same as Fig. 13 but for antiparallel bare helicity.

Yukawa term
3+1
DLCQ

little
recent
SS3

Anomalous Gravitomagnetic Moment $B(0)$

QED



$$F_2(0) = \frac{\alpha}{2\pi}$$

$$B(0) = \frac{\alpha}{3\pi} - \frac{\alpha}{3\pi} = 0$$

$$B(q^2) \sim \frac{\alpha}{\pi} \sqrt{q^2/m_c^2}$$

Equivalence Principle : $B(0) = 0$

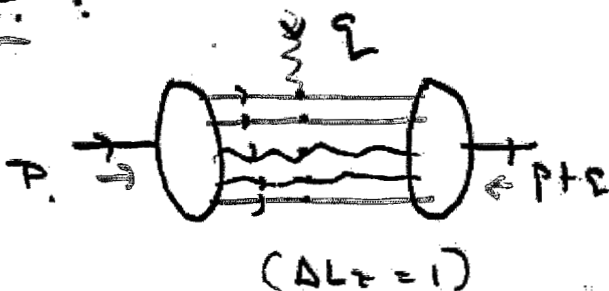
Okun + Kobzarev (62)
Xiao, Teryosov

any spin 1/2 system

L.F. :

lim
 $q^2 \rightarrow 0$

$$\sum_{i=1}^n$$



$$\sum_{i=1}^n B_i(0) = 0$$

Fock state
by Fock state

Result of Lorentz prop of LF wavefunction.

Hwang, Ma,
Schnitz, GJB

key Question for LG Th

$$B_p(0) = 0 ?$$

Important indicator of lattice errors.

Some advantages of Light-Front-Quantized QCD

- ✧ No Fermion doubling
- ✧ Multiple Fermion flavors
- ✧ Minkowski Space
- ✧ Gauge-Fixed : Physical Degrees of Freedom
- ✧ Manifest Frame-Independence : P^+ , \vec{P}_\perp arbitrary
 J_z kinematical
- ✧ LF Vacuum Trivial - Zero modes
- ✧ Vanishing anomalous gyromagnetic moment
 $B(0) \equiv 0$; $\lim_{A \rightarrow 0} \mu_A = 0$
- ✧ DLCQ discretization retains symmetries
 Continuum limit : $k \rightarrow \infty$
- ✧ LF Wavefunctions, continuum solns.,
 Amplitudes, Phases as well as spectra
- ✧ Solns for QCD(2+1), SUSY(2+1)
 QCD(3+1) : Challenging!

Light-Front Quantization of Standard Model SU(2) & U(1)

* $n \cdot A = A^+ = 0$ gauge

\Rightarrow { physical gauge quant
Unitary
renormalizable

* SSB from zero mode of scalar field

$$\phi^{(0)} = \frac{v}{\sqrt{2}}$$

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x) + i\eta(x))$$

$$\partial \cdot A = M \eta, \quad M = e v$$

* Goldstone field $\eta(x)$ restores E_L :

$$E_L = \frac{n_\mu M}{n \cdot k} - \frac{k_\mu M}{k^2} \quad (E \cdot k = 0)$$

* LC vacuum remains trivial

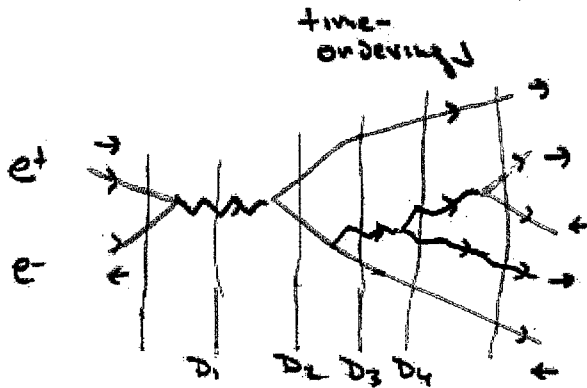
* Amplitude Event Generator

\Rightarrow Renormalized Amplitudes from LF T-O PTL
Ghost-Free, $\pi d^2k_\perp dx$, DLCQ discret.

"Event Amplitude Generator"

Generate amplitude from LF TO PTH $\left\{ \begin{array}{l} \text{Tree +} \\ \text{Virtual} \end{array} \right.$

$$M = \sum M_\alpha \quad (\text{specific spins } S_e)$$



$$M_\alpha = H_L \frac{1}{D_1} H_L \frac{1}{D_2} \dots$$

$\sum k^+, \sum k_\perp, \sum J_z$
conserved

$k^+ > 0$: few surviving LF time-orderings

Physical polarization sums : $\sum_{(i)} \epsilon_H^{(i)} \epsilon_V^{(i)}$

$(i) = 1, 2, 3$

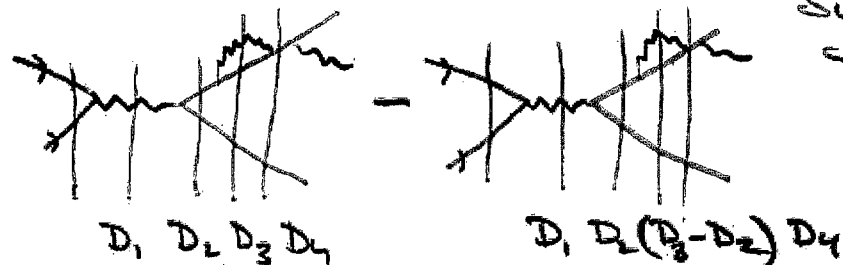
↑
Stueckelberg
model W

Compute renormalized amplitude

- "alternating denominator" method

Roskies
Suyke
JSB

Example :



equivalent to subtracting
mass counterterm!

$$\frac{1}{D_3 - D_2} = \frac{\delta m}{*}$$

$\pi d^4 k dx$, unitary, no ghosts.

Other Applications of LF Quantization

Light-Front Thermodynamics



Set boundary conditions at fixed $\tau = t + z/c$
not t

$$Z_{LF} = \sum_n \exp - \frac{m_n^2}{T_{LF}}$$

Light-Front Lippmann-Schwinger

$$T = H_E + H_E \frac{1}{m^2 - \sum \frac{m_i^2}{x} + i\epsilon} T$$

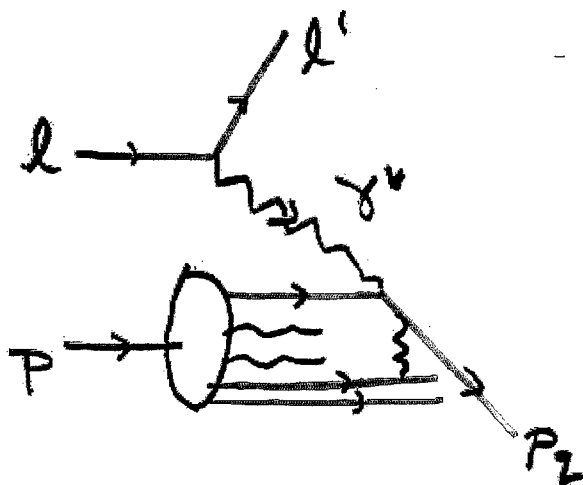
Variational Solutions to Bound-State Probs.

minimize $\langle \psi_{\text{trial}} | H_{LF} | \psi_{\text{trial}} \rangle$

Construct $\langle \psi_{\text{trial}} | h \rangle = \psi_h^{\text{trial}}(x_i, k_{zi}, \lambda_i)$

using numerals from L_2
Ladder relations

Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange
after photon ocb
not in LFWF

* Single-spin asymmetry

$$\vec{S}_p \cdot \vec{q} \times \vec{P}_q$$

Bjorken-scaling

* Diffraction at Leading Twist

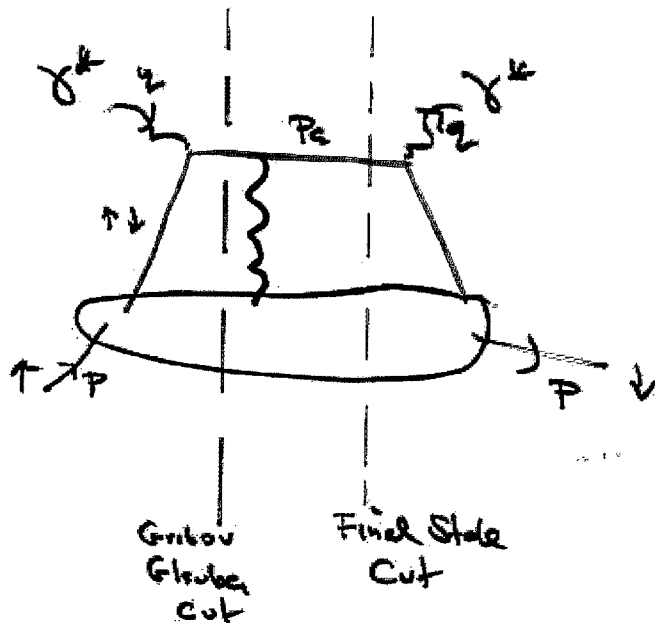
* Nuclear Shadowing (interference of diff channels)

* Energy Loss, P_T Broadening

Diffraction, Nuclear Shadowing, Pomeron
not in nuclear wavefunction!

Explicit calculation of FSI SSA I. A. Schmitt
SJB

hep/th/0201296



Overlap of
wavefunctions with
 $\Delta L_2 = 1$

$$[e^{i(\chi_1 - \chi_2)}]$$

χ_1, χ_2 : IR Finite

$$i \vec{S}_p \cdot \vec{q} \times \vec{P}_q = i \vec{S}_p \cdot \vec{q} \times \vec{r}$$

$$\vec{P}_q = \vec{q} + \vec{r}$$

$$P_y = A_n \approx \frac{\alpha_s(r_{\perp}^2) \times_{B_1} M |r_{\perp}^2| \ln r_{\perp}^2}{r_{\perp}^2}$$

Bjorken scaling for finite r_{\perp}

Some matrix elements as $\alpha_p = F_2(0)$

Conclusions

Important Role of FSI, ISI

- Leading Twist Diffraction in DIS
- Color Dipole Physics
- Nuclear Shadowing: not in nuclear LC w.f.!
- Single-Spin Asymmetries

{
 Bjorken Scaling!
 New spin-dep structure functions W_1', W_2', W_3'
 Nuclear Enhancement

{
 Shadowing / Anti-Shadowing
 Energy Loss
 PT Spreading
 Diffraction
 SSA

Begins
of fund. desc.
in QCD

Factorization, Local and Nonlocal Operators

George Sterman

C.N. Yang Institute for Theoretical Physics, SUNY Stony Brook, Stony Brook, NY 11793-3840

Hard-scattering cross sections for the production of a final state F characterized by a single hard scale M can be written in factorized form, as the convolutions of a perturbative function $\hat{\sigma}$ with parton distributions:

$$\sigma_{A+B \rightarrow C}(M) = \sum_{ijk} \phi_{i/A}^{(3)}(x, \mu_F) \otimes \phi_{j/B}(x', \mu_F) \otimes \hat{\sigma}_{i+j \rightarrow F(M)}(xp, x'p', M) \quad (1)$$

The parton distributions ϕ are determined in QCD by correlation functions with operators on the light cone,

$$\phi_{f/A}(x) = \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle P | \bar{q}_f(y^-) \left[P \exp \left(ig \int_0^{y^-} A^+(y^-) \right) \right] \Gamma q_f(0) | P \rangle ,$$

If the process has only a single hard scale, corrections to Eq. (1) are down by a power, and nonperturbative information is carried almost entirely in the parton distributions for these observables. Lattice calculations are able to provide first-principles calculations of low moments of the distributions [1], and progress has been made in this direction.

Processes with two scales: $M > m > \Lambda_{\text{QCD}}$ provide a wider variety of corrections. A well-studied example is the Drell-Yan cross section for lepton pairs of mass Q at fixed pair transverse momentum Q_T . In this case, we may factorize the cross section in terms of parton distributions at measured parton transverse momentum,

$$\mathcal{P}_{f/A}(x, k_\perp) = \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} e^{-ixP^+y^- + ik_\perp \cdot y_\perp} \langle P | \bar{q}_f(0^+, y^-, y_\perp) \Gamma q_f(y_1^-) | P \rangle ,$$

where the fields are now spacelike separated. At low Q_T , the cross section may be thought of as a convolution of resummed perturbative and nonperturbative functions. The nonperturbative component may be analyzed in impact parameter space, $W_{\text{NP}}(b) = \int \frac{d^2 b}{(2\pi)^3} e^{ib \cdot Q_T} \sigma_{\text{NP}}(Q_T)$. The logarithmic dependence on b of the function W_{NP} exponentiates [2] with a well-defined dependence on Q [3],

$$\ln W_{\text{NP}}(b) = \ln \left(\frac{Q}{Q_0} \right) \int_0^\infty \frac{d\sigma}{\sigma^2} \left[\Phi_1(i\sigma) f(i\sigma b^2) - 2\Phi_2(i\sigma) (1 - e^{-i\sigma b^2}) \right] .$$

Here the functions Φ can be determined on the lattice from nonlocal vacuum correlations at spacelike separations [4]. Considerations of this kind could lead to additional applications of lattice calculations to high energy cross sections.

References

- [1] QCDSF Collaboration (S. Capitani et al.), 19th International Symposium on Lattice Field Theory (Lattice 2001), Berlin, Germany, 19-24 Aug 2001, in Nucl. Phys. Proc. Suppl. 106, 299 (2002), hep-lat/0111012.
- [2] G.P. Korchemsky and G. Sterman, Nucl. Phys. B437 415 (1995), hep-ph/9411211.
- [3] S. Tafat, JHEP 0105:004 (2001), hep-ph/0102237.
- [4] A. Di Giacomo and H. Panagopoulos, Phys. Lett. B285, 133 (1992).

Matrix elements

$$i = f, s$$

$$\phi_{q_f, s/A}(x, \mu^2) = \int \frac{dy^-}{2\pi} e^{-ixp^+ y^-}$$

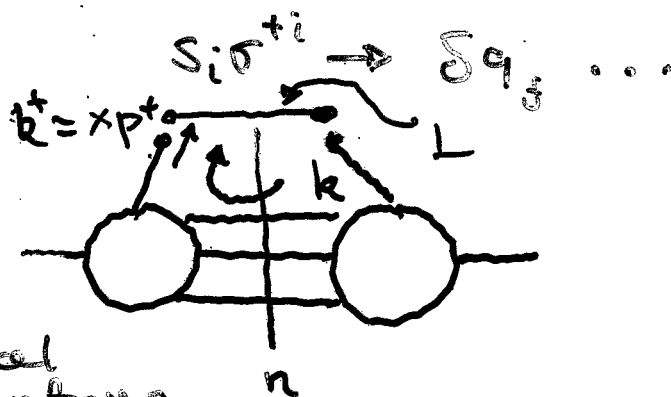
$$\langle A(p, s) | \bar{q}_f(y^-) \Gamma_{\gamma i \alpha}^{(s)} q_f(0) | A(p, s) \rangle$$

$$L(y^-, 0) = P e^{ig \int_0^{y^-} dx A^+(x)}$$

$$x^+ \rightarrow q_f(x)$$

$$\delta^+ x^5 \rightarrow \Delta q_f$$

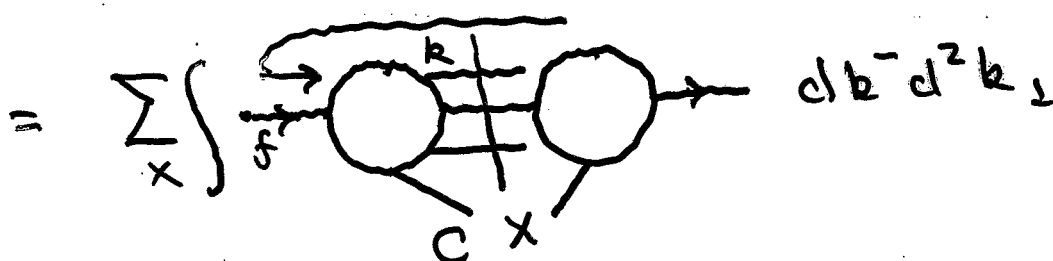
$$= \sum \int_{k_\perp^2 < \mu^2} dk^- d^2 k_\perp$$



Moments $mx \rightarrow$ local operators

$$D_{C/q_f, s}(z, \mu^2) = \int \frac{dy^-}{2\pi} e^{-i(p_c^+/z)y^-}$$

$$\times \langle 0 | q_f(0) | Cx \rangle \Gamma_{(s) \gamma}^{(s) L} \langle Cx | \bar{q}_f(y^-) | 0 \rangle$$



FACTORIZATION WITH k_{\perp} -UNINTEGRATED DISTRIBUTIONS

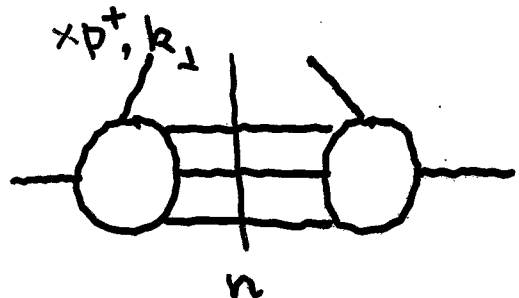
- distributions: (physical gauge)

Collins + Soper 81

see below for p^+ -dependence

$$\mathcal{P}_{f,S}(x, k_{\perp})$$

$$= \sum_n \int dk^-$$



$$= \int \frac{dy^- d^2 y_{\perp}}{(2\pi)^3} e^{-ixp^+ y^- + i \underline{k}_{\perp} \cdot \underline{y}_{\perp}}$$

$$\cdot \langle A(p, S) | \bar{q}_f(0^+ y^- y_{\perp}) \Gamma^{(S)} q(0) | A(p, S) \rangle$$

$$\phi_{f,S}(x, \mu) = \int d^2 k_{\perp} \mathcal{P}_{f,S}(x, k_{\perp})$$

$$\mathcal{D}_{f,S}(z, k_{\perp}) = \sum_x \int dk^-$$

- gauge dependence cancels in cross sections

Factorization at 'fixed k_T '

k_T -dependence power-suppressed unless

two observables

$$M \gg m \gtrsim \Lambda_{QCD} (!)$$

Examples:

- $A+B \rightarrow V(Q, Q_T) + X$

$$\frac{d\sigma}{dQ^2 d^2Q_T} \sim \mathcal{P}_{i/A}^{Q \gg Q_T} \otimes \mathcal{P}_{j/B}^{Q_T \gg m} \otimes \hat{\sigma} \delta^2(Q_T - k_T - k'_T) + Y$$

$\uparrow \sim \frac{1}{Q_T^2}$
 $\uparrow \sim Q_T^0$

- Collins effect

$$\frac{d\sigma(s_\perp)}{dp_H dp^{Jet}} \sim h(x, s_\perp) \otimes \hat{\sigma} \otimes \underbrace{\langle s \cdot p_H \times p_J \rangle}_{\langle |p_H \times p_J| \rangle}$$

notation heuristic!

$$M \sim p_H \text{ or } p^{Jet}$$

$$m^2 \sim p_H \times p_J$$

- e^+e^- : Collins Soper & ... Boer 00

SOME THINGS WE KNOW

- $$\mathcal{P}(x, k_T) \xrightarrow{k_T \rightarrow \infty} \int_x^1 d\xi \phi(\xi, \mu) \cdot C\left(\frac{x}{\xi}, \frac{k_T}{\mu'} \alpha_s(\mu)\right)$$

$$\sim \frac{\alpha_s(k_T^2)}{k_T^2}$$

- \mathcal{P} depends on p^+ in $\langle A(p) \rangle$

\hookrightarrow suppression at small- k_\perp
large- b

$$\mathcal{P}(x, k_T) \sim \int d^2b e^{ik_T \cdot b} e^{-S(b, Q)}$$

$Q \leftarrow$ actually $p_{\perp \text{ gauge}}$

$$S(b, Q) \sim c \int_{1/b}^{\mu'} \frac{d\mu'}{\mu'} \ln \frac{\mu'}{Q} \alpha_s(\mu') + \dots$$

- spin-independent
- the larger p^+ , the more initial-state radiation
- but in single-scale cross sections only contributes to higher orders in $\hat{\sigma}$

Vacuum vs. Land Hadroneic k_T

$$\ln W(b, Q) \sim 2 \ln \mathcal{P}(b, Q)$$

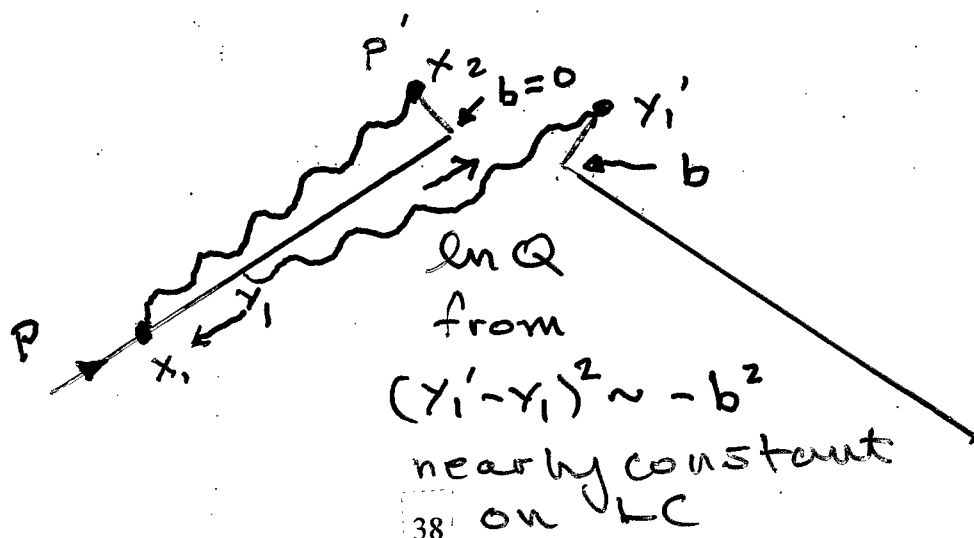
$$\sim S_{PT}(b, Q) + S_{NP}(b, Q)$$

$$\int_{b_0^2/b^2}^{Q^2} \frac{dk^2}{k^2} \left[A(\alpha_s(k^2) \ln \frac{Q^2}{k^2} + B(\alpha_s(k^2)) \right]$$

$$\begin{aligned} & \phi(b) \ln \frac{Q}{Q_0} \\ & + \phi_{q/p}(b, x) \\ & + \phi_{\bar{q}/p}(b, x') \end{aligned}$$

- $\phi(b)$ 'vacuum'
- $\phi_{q/p}(b, x)$ 'intrinsic'

$\ln Q$ in PT and NP of same origin



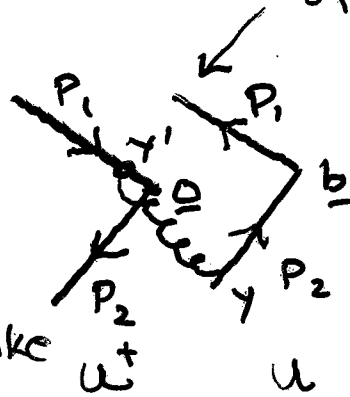
S. Tafat '01

Vacuum $k_T \leftrightarrow$ Field strength
Correlators at
Spacelike distances

$$\sigma_{eik} = \sum_N |\langle 0 | U_{DY}^{(0)} | N \rangle|^2 \delta^{(2)}(\underline{k}_N - \underline{q})$$

$$= \int \frac{d^2 b}{(2\pi)^2} e^{-i \underline{b} \cdot \underline{q}} W_{DY}(b)$$

$$W_{DY}(b) = \langle 0 | P \exp \left(i g \int_{C_{DY}} dz \cdot A(z) \right) | 0 \rangle$$



$(y-y')^2$ always spacelike
nearly constant

$$\ln W \sim \ln \frac{Q}{Q_0} \int_0^\infty \frac{d\sigma}{\sigma^2} \left[\tilde{\Phi}_1(i\sigma) f(i\sigma b^2) - 2 \tilde{\Phi}_2(i\sigma) (1 - e^{-i\sigma b^2}) \right]$$

$$f(x) = e^{-x} - 1 + \sqrt{\pi x} \operatorname{erf}(\sqrt{x})$$

Dosch
Shcherchenko
Simonov $\tilde{\Phi}_{1,2}$ from nonlocal vacuum corr.

$$\langle 0 | g^2 F(x) [P e^{i g \int_{\gamma} A}] F(0) | 0 \rangle$$

Chiral Aspect of Parton Physics

Xiangdong Ji (University of Maryland)

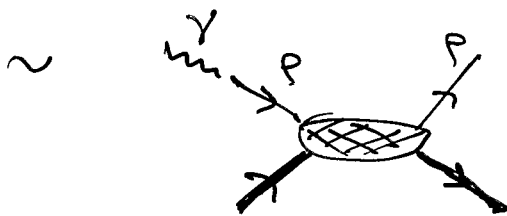
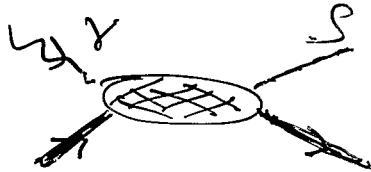
1. Meson cloud models of the nucleon has been used in the last thirty years to explain the electromagnetic form factors of the nucleon (particularly neutron charge radius), the strange content, the sea quark distributions, the EMC effects (modifications of the parton distributions in the nuclear environment) etc. However, the theoretical basis of the description has been unclear.
2. We have found an effective field theory approach to understand the meson cloud structure of the nucleon. In particular, we now have a formalism to study the quark distributions in the nucleon through the meson cloud contributions. The approach starts from the twist-two operators of QCD. We match these operators to the operators of the same quantum numbers in the chiral effective theory. The matching process is an Operator Expansion that separates the Goldstone boson physics from physics at hadron mass scale.
3. Using the above approach, we have calculated
 - a. The leading chiral contribution to the difference between up and down quark seas in the nucleon.
 - b. The leading chiral contribution to the unpolarized, parton helicity, transversity distributions.
 - c. The leading chiral contribution to the spin structure of the nucleon.
 - d. The leading chiral contribution to the gravitational form factors of the nucleon
4. The above results are useful to
 - a. Extrapolate the lattice QCD calculations at large quark masses to physical ones.
 - b. Understand the chiral structure of the nucleon.

Hadronic Structure of the Photon

With Chulwoo Jung
PRL, 2000

- $U(1)$ gauge boson, asymptotic physical state in the Coulomb phase of QED
- Couple with quarks, develop nontrivial strong interaction structure

Classical Example:



VECTOR DOMINANCE
MODEL

However, the photon structure functions cannot be computed in lattice QCD in the same way as for the hadron structure functions.

Recall

$$\langle 0 | \chi_H(y) \hat{O}(x) \bar{\chi}(0) | 0 \rangle \underset{T_y, T_x \rightarrow \infty}{\sim} e^{-T_y M_H} \langle H | \hat{O} | H \rangle$$

The obvious interpolating field $\bar{\chi}_\sigma \sim \sum_g \bar{q} \gamma_\sigma q$ does not create a photon state through Euclidean-time filtering.

Instead, it creates vector mesons if they are stable, and $\pi\pi$ P-wave scattering states at threshold if not.

Photon is not an asymptotic state of QCD.

Consider QED + QCD on a lattice

$A^\mu(x)$ then is the right interpolating field to create a physical photon state

$$\langle x | \hat{O} | x \rangle \sim \langle 0 | A_\mu(y) \hat{O} A_\nu(0) | 0 \rangle \epsilon_\mu^\dagger \epsilon_\nu$$

However, QED is perturbative and the photon field integration is trivial

$$\begin{aligned} & \langle x(p\lambda) | \hat{O} | x(p\lambda) \rangle \\ &= -e^2 \int d^4x d^4y e^{i\omega(x_4 - y_4)} e^{-i\vec{p} \cdot (\vec{x} - \vec{y})} \\ & \quad \langle 0 | T_E \epsilon^\dagger(\lambda) \cdot J(x) \hat{O}(0) \epsilon(\lambda) J(y) | 0 \rangle \end{aligned}$$

- T_E : Euclidean time-ordering
- $J_\mu = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f$

$$\gamma_i = -i\gamma^i_{EM}, \quad \gamma_4 = \gamma^0$$

$$\epsilon_i = \epsilon^i_M, \quad \epsilon_4 = i\epsilon^0 \quad (\epsilon_4^\dagger = i\epsilon_0^\dagger)$$

Leading chiral logs in the flavor asymmetry of the Sea

$$\begin{aligned} \int_0^1 (\bar{d}(x) - \bar{u}(x)) dx &= \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} \\ &+ \int_0^1 (\bar{d}(x) - \bar{u}(x))^0 dx \left(1 - \frac{3g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} \right) \\ &+ \dots \end{aligned}$$

- Exact result of QCD !
- Disagree with A.W. Thomas et al.
PRL 85, 2892 (2000)

the usual meson cloud calculations
do not have right chiral properties.

There are other contributions at
the same order.

- The first term is negative !

Chiral contribution is not the
whole story !

Using the above formalism, we have obtained the leading chiral contribution to the moments of

- Unpolarized Quark Distributions

$$\langle x^n \rangle_{ud} = C_n \left\{ 1 - \frac{(3g_A^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln \frac{m_\pi^2}{\mu^2} \right\} + \dots$$

- Quark Helicity Distributions

$$\langle x^{n-1} \rangle_{\Delta u - \Delta d} = \tilde{C}_{n-1} \left\{ 1 - \frac{(2g_A^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln \left(\frac{m_\pi^2}{\mu^2} \right) \right\} + \dots$$

- Quark Transversity Distribution

$$\langle x^{n-1} \rangle_{\delta u - \delta d} = \tilde{C}_{n-1} \left\{ 1 - \frac{4g_A^2 + 1}{2(4\pi f_\pi)^2} m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \right\} + \dots$$

Transversity measurements at RHIC and KEK

OGAWA, Akio

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Upton, NY, 11973-5000, U.S.A*

The transversity is the last missing part among the 3 quark distributions at the leading twist. While helicity average and difference distributions were well measured by DIS, helicity flip distribution is harder to access due to its chiral odd nature. To complete measurements of transversity, few different processes have to be measured in polarized proton collisions (BNL-STAR/PHENIX), polarized DIS (DESY-Hermes, CERN-Compass) and electron positron collisions (KEK-Belle).

From polarized proton collision double transverse spin asymmetries A_{TT} of Drell-Yann or di-jets measurements gives unique direct measurements of transversity. But the asymmetries are small, and only future RHIC upgrade may enable us to reach this channel.

Both polarized proton collisions and also polarized DIS have access to transversity through looking at azimuthal asymmetries in jet fragments [5] [6], if there is non-zero chiral odd spin dependent fragmentation functions, which is referred as Heppelmann-Collins effects [2]. In unpolarized electron positron collisions [1], one can measure these chiral odd fragmentation functions, such as the Heppelmann-Collins function C [2] or two pion interference fragmentation function q_T [3] [4]. Combining these results will give a measurement of Transversity. A small group of people from RHIC spin, including Matthias G. Peedekamp and myself, was accepted to the Belle collaboration to carry out the extraction of these fragmentation functions.

It also has been discussed [9] that the single spin asymmetries A_N measured by FermiLab E704 [7] and AGS E925 [8] can be understood in terms of the Collins effect. At the first polarized proton run at RHIC, STAR and PHENIX have started a measurements of A_N in different kinematics regions, and results are to be come out.

References

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- [2] J.C.Collins, Steve F.Heppelmann, G.A.Ladinsky, Nucl. Phys. B396 :161, 1993
- [3] R.L.Jaffe et al., Phys. Rev. Lett. 80 :1166, 1998; Boffi, R. Jakob, M. Radici hep-ph/9907374
- [4] A. Bianconi, S.Boffi, D. Boer, R. Jakob, M. Radici, hep-ph/0010132
- [5] HERMES collaboration, Phys. Rev. Lett.84:4047, 2000
- [6] SMC collaboration, Nucl. Phys. Proc.Suppl.79:520,1999
- [7] Fermilab E704 Collaboration, Phys. Rev. Lett.77: 2626, 1996; Phys. Rev. D53 :4747, 1996
- [8] AGS E925 collaboration, Phys. Lett. B459 :412, 1999
- [9] M. Anselmino, M. Boglione, F. Murgia, Phys. Rev. D60 :054027, 1999

“Complete” Transversity Measurements

Polarized pp - RHIC Star/Phenix/Phobos (BNL)

$$\pi^+ \pi^- \text{ Interference Fragmentation : } A_T(p_\perp + p \rightarrow \text{jet}(\pi^+, \pi^-) + X) \Leftrightarrow \delta q \cdot \delta \hat{q}_I$$

$$\text{Collins Effect : } A_T(p_\perp + p \rightarrow \text{jet}(h) + X) \Leftrightarrow \delta q \cdot C$$

$$\text{Drell Yan : } A_{TT}(p_\perp p_\perp \rightarrow ll) \Leftrightarrow \delta q \cdot \delta \bar{q}$$

$$\text{Inclusive hadron : } A_N(p_\perp p \rightarrow h) \Leftrightarrow \delta q \cdot C + 2 \text{ other terms}$$

Polarized DIS - Hermes(DESY) Compass(CERN) eRHIC Tesla-N

$$\text{Collins Effect : } A_T(lp_\perp \rightarrow l + \pi + X) \Leftrightarrow \delta q \cdot C$$

$$\pi^+ \pi^- \text{ Interference Fragmentation : } A_T(lp_\perp \rightarrow \text{jet}(\pi^+, \pi^-) + X) \Leftrightarrow \delta q \cdot \delta \hat{q}_I$$

e+e- collider - Belle (KEK) Babar LEP ...

$$e^+ e^- \rightarrow \text{dijet} : C \cdot C, \delta \hat{q}_I \cdot \delta \hat{q}_I \text{ \& } C \cdot \delta \hat{q}_I$$

Tensor Charge Lattice calculations - RBRC

$$\delta \Sigma = \delta u + \delta d + \delta s = 0.56 \pm 0.09$$

S. Aoki, M. Doui, T. Hatsuda and Y. Kuramashi Phys.Rev. D56 (1997)433
More recently: S. Capitani et.al. Nucl. Phys. B (Proc. Suppl.) 79 (1999) 548

Belle experiment at KEK

$E = 10.58 \text{ GeV}$

$\sim 60 / \text{fb}$ or $\sim 10^8$ Events on tape

$\sim 360 / \text{pb} / \text{day}$

peak $L = 6.721 \cdot 10^{33} / \text{cm} / \text{cm} / \text{s}$

World Record!

design $L = 10.0 \cdot 10^{33} / \text{cm} / \text{cm} / \text{s}$

(Babar = $3.3 \cdot 10^{33} / \text{cm} / \text{cm} / \text{s}$)

The world best statistics

Hermetic detector

Excellent PID upto top energy

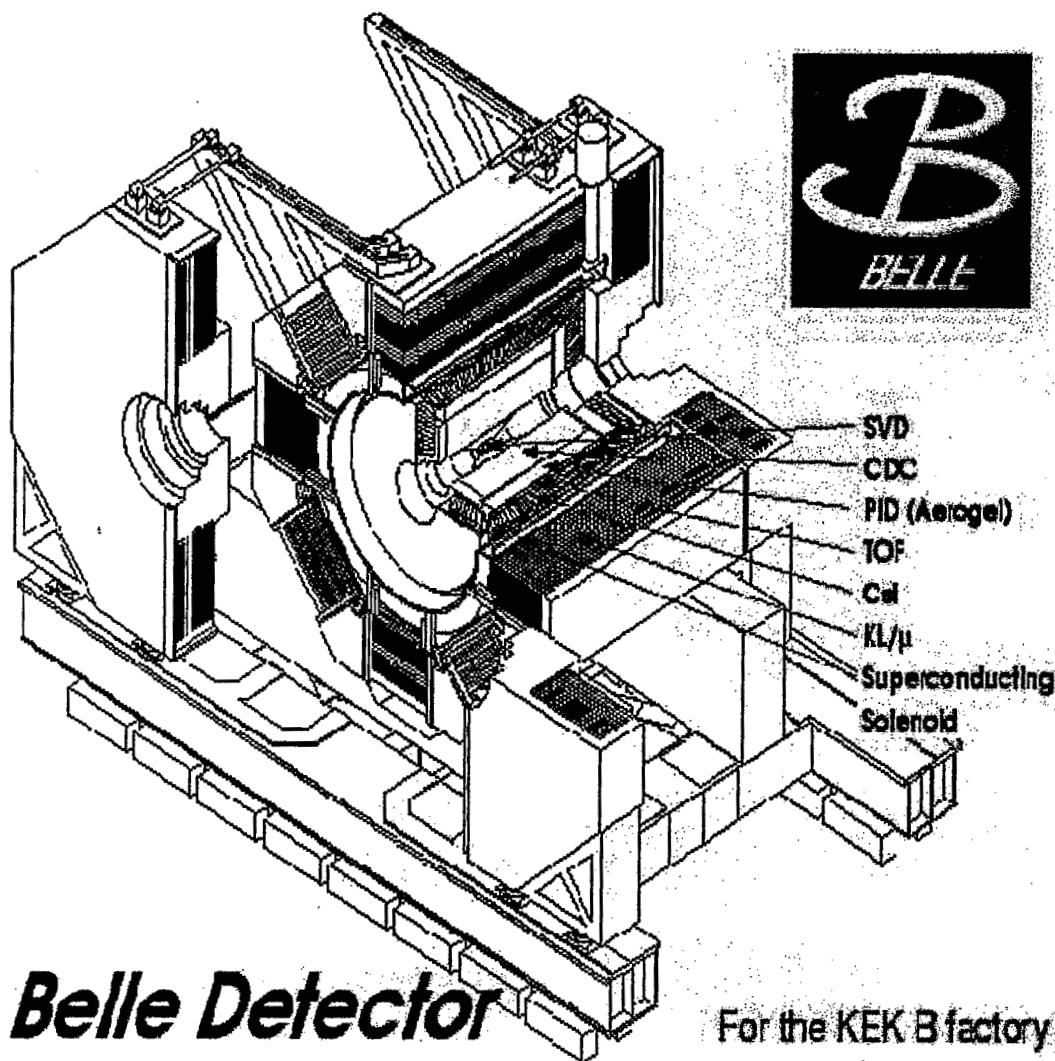
High enough energy

$\sqrt{s} \sim 100 \text{ GeV}^2$

Not too high energy

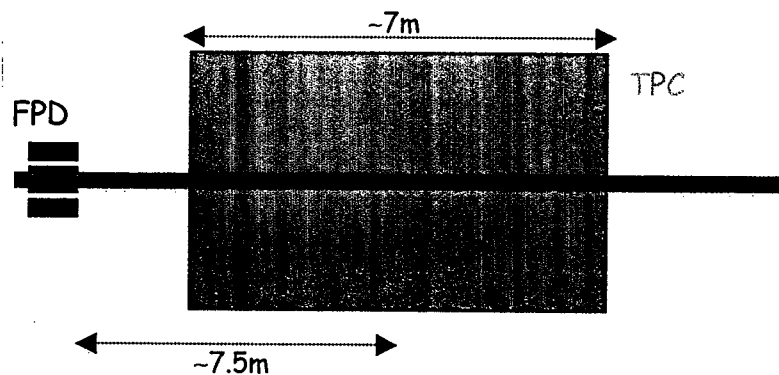
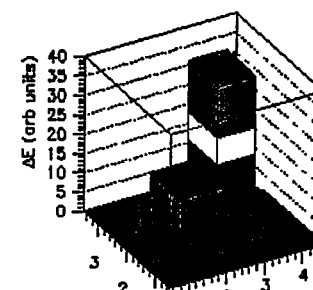
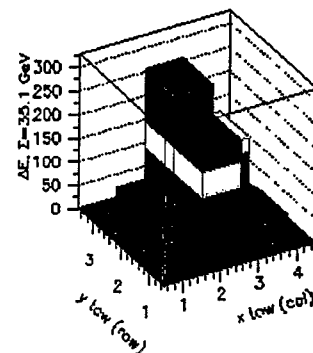
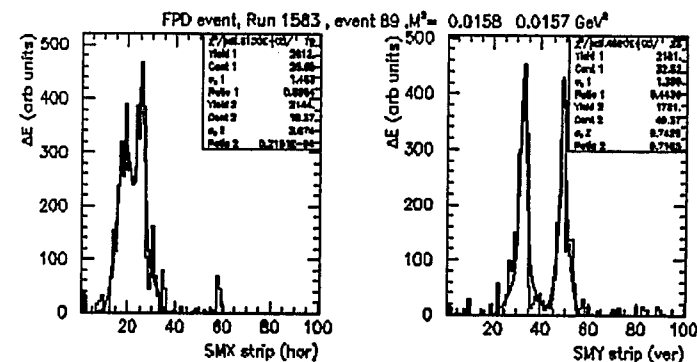
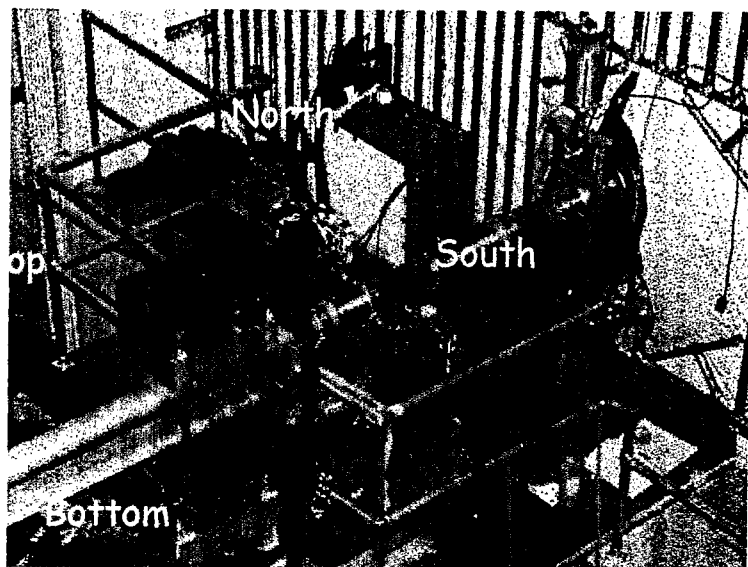
- below Z pole

- FF evolves down as energy goes up



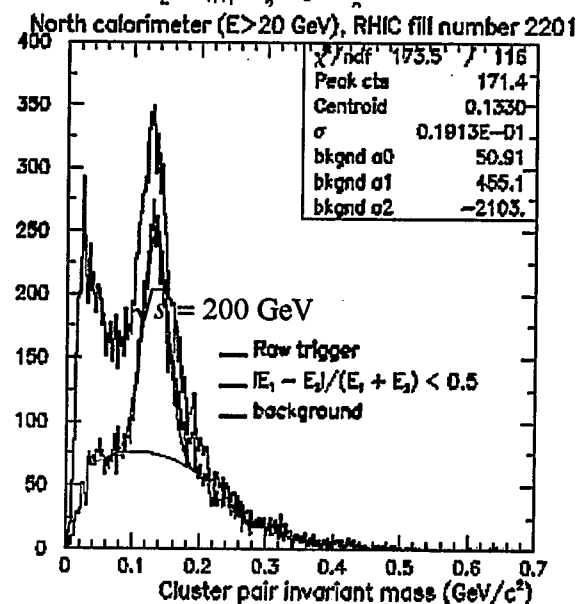
RBRC group (M.G-Perdekamp, S. Lange and A.Ogawa) have joined Belle

STAR Forward Pion Detector



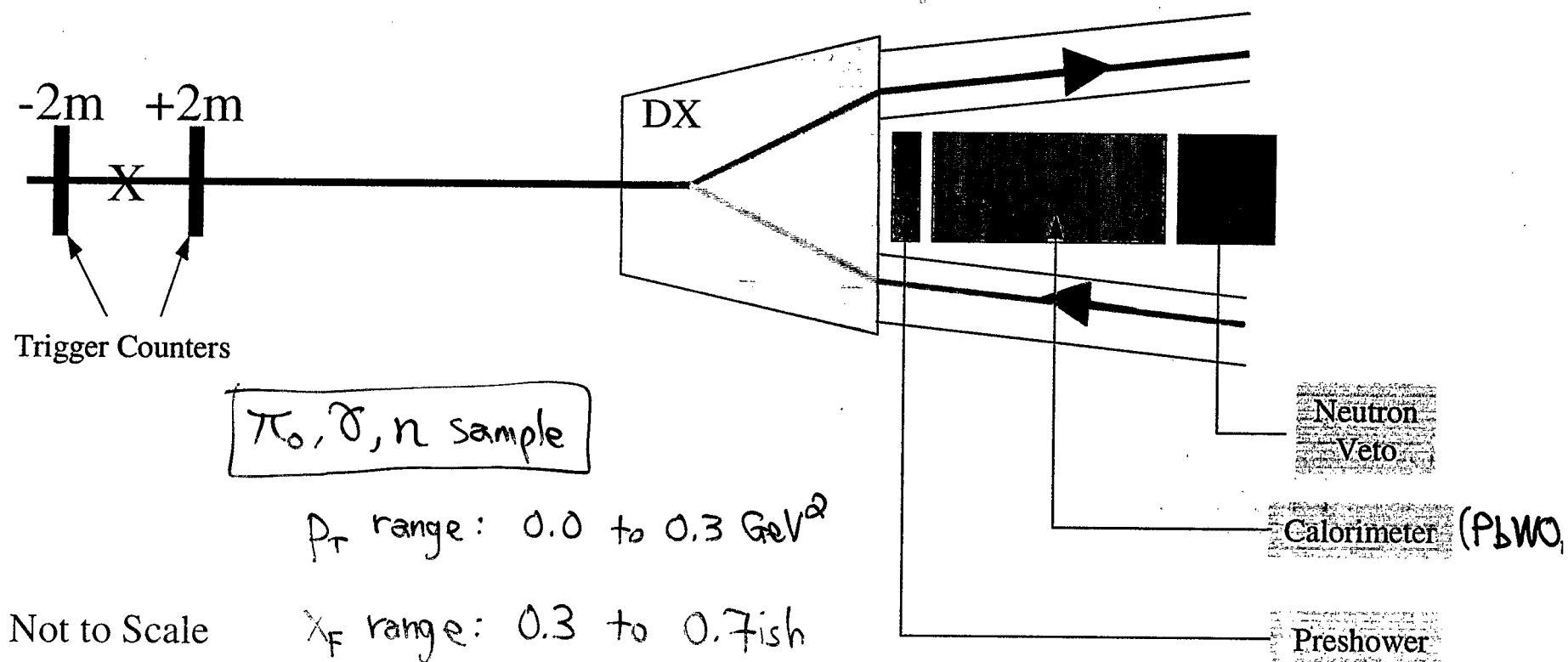
$X_F \sim 0.1 \text{ to } 0.6 \text{ or } -0.1 \text{ to } -0.6$
 $p_T \sim 1 \text{ to } 4 \text{ GeV}$
 $E \sim 10 \text{ to } 60 \text{ GeV}$

Online Results :
 π^0 reconstruction
 up to 60 GeV



Schematic of the Experiment

@ IP12



Conclusion

- RHIC is the first polarized pp collider
 - Just finished the first successful run
 - Single spin asymmetries A_N are on the way out
 - at STAR FPD (and also mid rapidity)
 - at 12 o'clock zero degree
 -
- RHIC (and pol DIS) will measure Transversity
 - with fragmentation function from e^+e^- (Belle)
 - with A_{TT} from DY / Jets (Luminosity upgrade needed)
- Belle will extract Heppelmann-Collins FF and Interference FF
 - RHIC spinners are working on it!
- Lattice calculations of tensor charge

Helicity Distributions at RHIC

Yuji Goto

RIKEN BNL Research Center

We measure the gluon helicity distribution and flavor-decomposed quark helicity distributions at RHIC. This is done by measuring the double longitudinal-spin asymmetry (A_{LL}) and parity-violating single longitudinal-spin asymmetry (A_L) with longitudinally polarized proton collisions. In the last polarized proton collision run (2001–2002), we performed just transversely polarized proton collisions. The longitudinally polarized proton collisions will be performed in the next polarized proton collision run.

One big motivation of the helicity distribution measurements at RHIC is so-called “spin crisis” which was found by polarized DIS experiments. In their measurements with polarized lepton beams on polarized nucleon targets, the quark spin contributes only 10–20% of the proton spin 1/2. The remaining fraction must be explained by the gluon spin and the orbital motion. The DIS experiments measure just the quark contribution in the leading order. The contribution of gluon is measured by using next-to-leading order processes, and it shows large uncertainties.

The polarized proton collisions at RHIC enable us to measure the gluon contribution to the proton spin 1/2 with the leading order processes like the gluon Compton process in the prompt photon measurement or the gluon fusion process in the heavy-flavor production measurement. By using various channels of the gluon helicity distribution measurement, the RHIC polarized proton collision program shows excellent performance with wide x -range coverage.

The flavor decomposition of the quark helicity distribution will be performed at RHIC with W^\pm production as a major channel. This has no fragmentation ambiguity, although the x -range is limited. It is a complementary measurement to the semi-inclusive DIS measurement which covers wide x -range but shows limited sensitivity to the flavors.

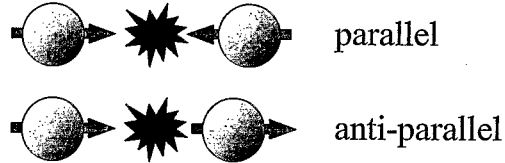
Helicity distributions at RHIC

- By A_{LL} or A_L measurement
 - double longitudinal-spin asymmetry

$$A_{LL} = \frac{d\sigma_{++} - d\sigma_{+-}}{d\sigma_{++} + d\sigma_{+-}}$$

- parity-violating asymmetry

$$A_L = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-}$$



- with longitudinally polarized proton collisions
- run3 (2002-2003) at RHIC !?

➔ Gluon helicity distribution

➔ Flavor-decomposed quark helicity distribution

- x -distribution in limited x -region

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Yuji Goto (RBRC)

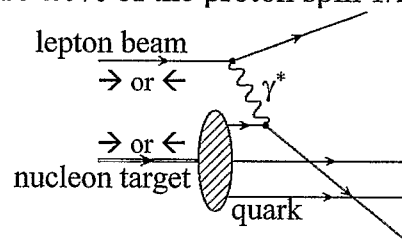
2

Proton spin 1/2

- Proton spin 1/2 by polarized DIS experiments
 - the quark helicity distribution contributes only 10-20% of the proton spin 1/2

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L$$

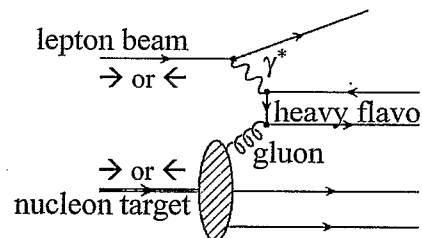
$$\Delta\Sigma = 0.1 \sim 0.2$$



- contribution of the gluon helicity distribution – next-to-leading order
 - Q^2 -evolution (global analysis by SMC group)

$$\Delta g = 1.0^{+1.0}_{-0.3}(\text{stat})^{+0.4}_{-0.2}(\text{sys})^{+1.4}_{-0.5}(\text{th}) \quad (Q^2 = 1 \text{ GeV}^2)$$

- photon-gluon fusion process
 - HERMES, COMPASS



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Proton spin 1/2

- Proton spin 1/2 by polarized proton collisions

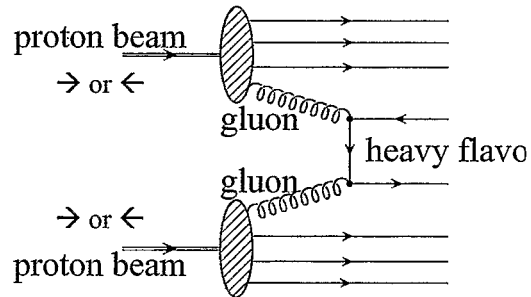
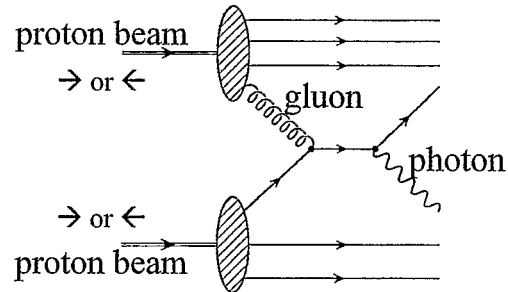
- contribution of the gluon helicity distribution – leading order

- prompt photon production – gluon Compton

$$gq \rightarrow q\gamma$$

- heavy flavor production – gluon fusion

$$gg \rightarrow c\bar{c}, b\bar{b}$$



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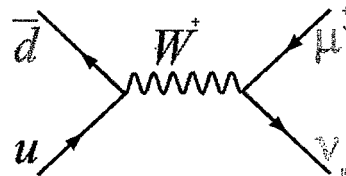
Proton spin 1/2

- Proton spin 1/2 by polarized proton collisions

- contribution of the flavor-decomposed quark helicity distribution

- W production

- decomposition selected by chirality



- neutral/charged pions

- decomposition weighted by the fragmentation function

- other contributions

- orbital angular momentum
- higher-twist effect
- k_T effect
- transversity
- ...

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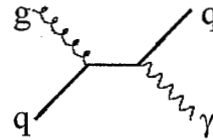
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Gluon helicity distribution

- Prompt photon production

- gluon Compton process dominant

$$gq \rightarrow q\gamma$$



- clear interpretation

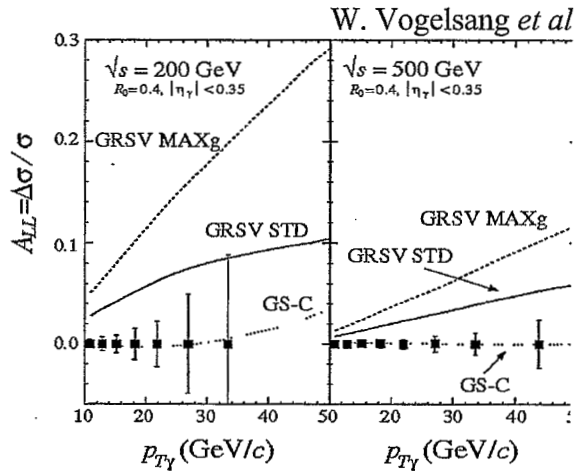
- $\Delta g(x_g)$ – gluon polarization measurement in the polarized proton collision

- double longitudinal-spin asymmetry

$$A_{LL}(p_T) = \frac{\Delta g(x_g, Q^2)}{g(x_g, Q^2)} \cdot A_1^p(x_q, Q^2) \cdot a_{LL}(\cos \theta^*)$$

$$A_1^p(x_q, Q^2) = \frac{g_1^p(x_q, Q^2)}{F_1^p(x_q, Q^2)} = \frac{\sum_i e_i^2 \cdot \Delta q_i(x_q, Q^2)}{\sum_i e_i^2 \cdot q_i(x_q, Q^2)}$$

$$i = u, \bar{u}, d, \bar{d}, s, \bar{s}, \dots$$



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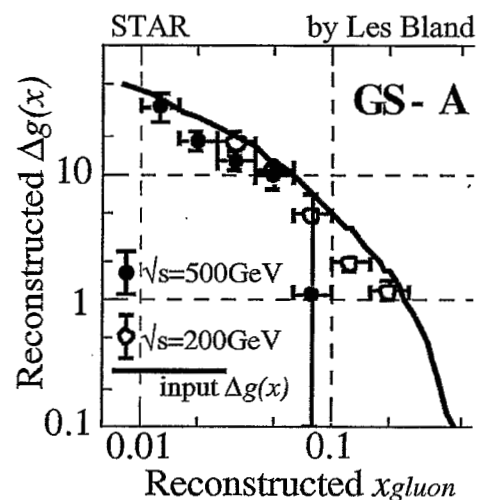
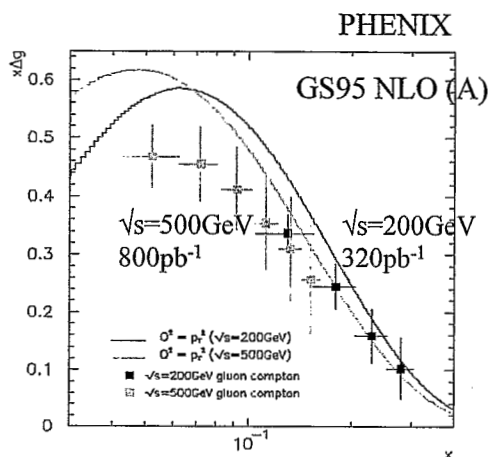
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Gluon helicity distribution

- Prompt photon production

- PHENIX inclusive γ
- STAR γ +jet

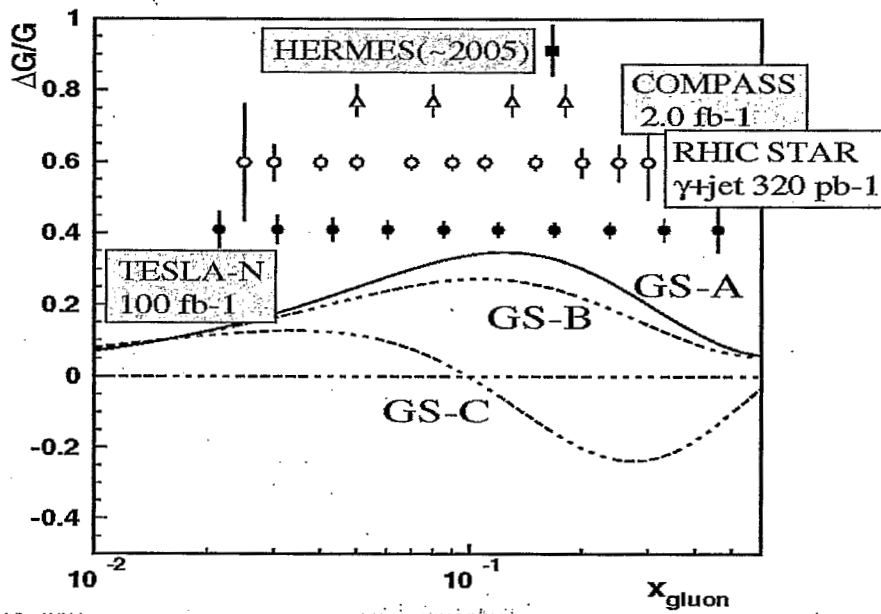


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Gluon helicity distribution

- RHIC spin is the best measurement among currently running experiments, even only with 1 exp. & 1 chan.



March 10, 2002

Yuji Goto (RBRC)

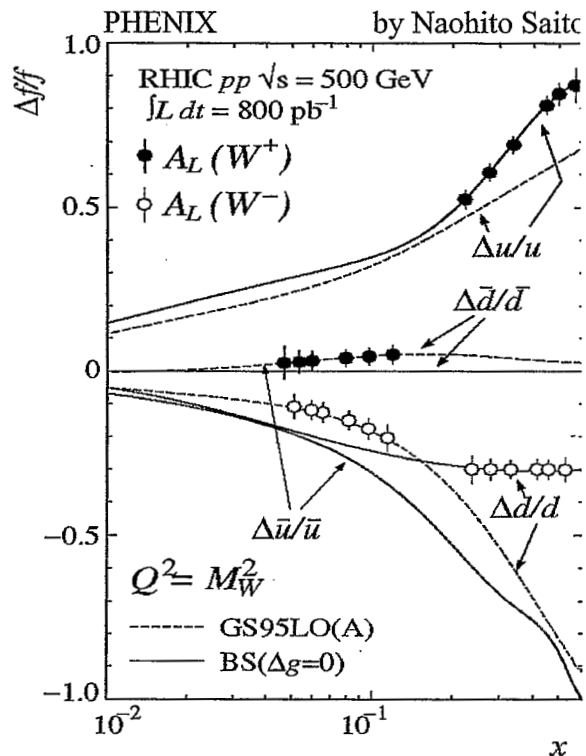
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Flavor-decomposed quark helicity distribution

- W production
 - parity-violating asymmetry A_L

$$A_L^{W^+} = \frac{\Delta u(x_a)\bar{d}(x_b) - \Delta\bar{d}(x_a)u(x_b)}{u(x_a)\bar{d}(x_b) + \bar{d}(x_a)u(x_b)}$$

- PHENIX Muon Arms
- STAR Endcap Calorimeter provides similar sensitivity
- $A_L \sim \Delta u/u(x) \sim 0.7-0.9$ at large- x

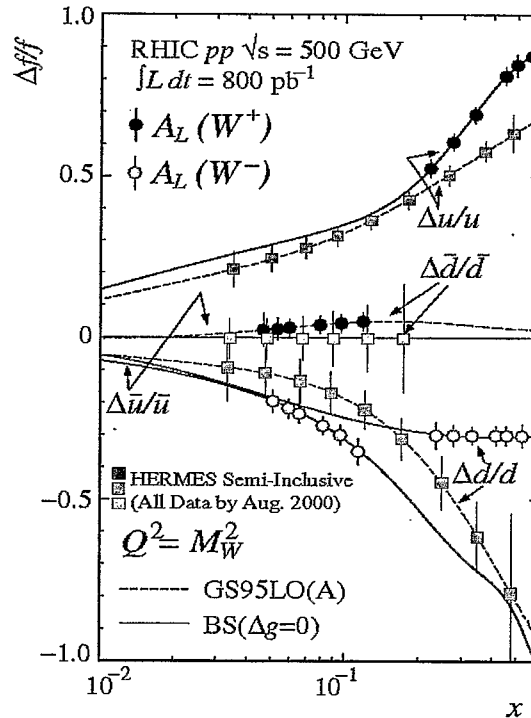


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Flavor-decomposed quark helicity distribution

- W production
 - no fragmentation ambiguity
 - x -range limited
- complementary to HERMES semi-inclusive DIS
 - wide x -range
 - limited sensitivity to sea flavors



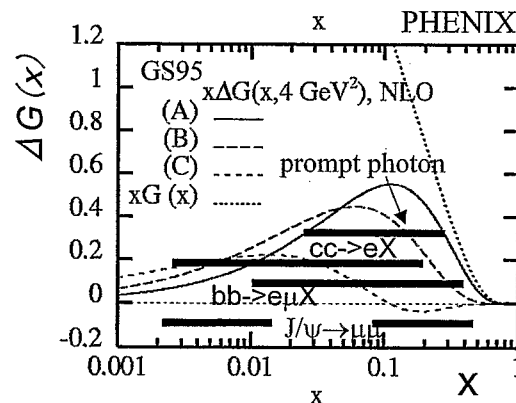
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Summary

- Helicity distributions at RHIC
 - gluon helicity distribution
 - prompt photon, photon+jet
 - jet, jet+jet
 - heavy flavor – electron, muon, e-μ coincidence
 - flavor-decomposed quark helicity distribution
 - W production
 - neutral/charged pions/hadrons
- Longitudinally-polarized proton collisions in run3 (2002-2003) !?



March 18, 2002

Yuji Goto (RBRC)

Chiral Corrections to the Nucleon Parton Distributions

Jiunn-Wei Chen

Parton distribution functions are fundamental properties of the hadron, however, currently they can only be computed directly from QCD on Euclidean lattices. On lattice simulations, due to the limitation of computing resources, quark masses heavier than their physical values and quenched approximations are usually employed. In this talk, I address the issues of chiral extrapolations and the quenching effects in hadronic parton distributions using effective field theory approaches.

The moments of nucleon parton distributions are related to matrix elements of local operators (so called “twist-2” operators) in QCD and can be simulated on lattices, at least for the first few moments. To study the quark mass dependence of these matrix elements to aid chiral extrapolations, chiral perturbation theory (χPT) is a natural tool to use. χPT is an effective field theory of QCD. It captures the symmetries and the symmetry breaking pattern of QCD around the zero quark mass limit and expands physical quantities in power series of $\sim m_\pi/(1\text{GeV})$. The machinery of computing matrix elements of local operators is well developed in χPT . Xiangdong Ji and I have used this approach to compute non-analytic quark mass dependence to several twist-2 matrix elements (see the following slides and Xiangdong’s talk).

The quenched and partially quenched effects usually used in lattice simulations can be studied using the corresponding effective field theories called quenched and partially quenched χPT ’s. Martin Savage and I have worked out the corresponding non-analytic quark mass dependence relevant for parton distribution function simulations (see the following slides and Martin’s talk).

Finally I discuss the connection between the quenched approximation and the $1/N_C$ expansion. Again, using the effective field theory approach, I found that in the mesonic sector, some low energy constants of the quenched χPT are the same as those in the unquenched case in the large N_C limit.

J_q of Proton (Chen + Ji, PRL, '02)

$$T^{\mu\nu} = T_q^{\mu\nu} + T_G^{\mu\nu}$$

$$= \frac{1}{2} \bar{\psi} \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} \psi + \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F^\nu{}_\alpha$$

$$\langle p' | T_q^{\mu\nu} | p \rangle$$

$$= \bar{u}(p') [A_q(q^2) \gamma^{(\mu} \bar{p}^{\nu)} + B(q^2) \bar{p}^{(\mu} i \sigma^{\nu)\alpha} q_\alpha / 2M + C_q q^{(\mu} q^{\nu)} / M] u(p)$$

where $\bar{p} = \frac{p+p'}{2}$, $q = p - p'$

$$\underbrace{J_q}_{\text{orbital}} = \frac{1}{2} [A(0) + B(0)] = \underbrace{L_q}_{\text{orbital}} + \underbrace{\Sigma_q}_{\text{helicity}}$$

J_q & Σ_q can be measured in lab

$\Rightarrow L_q$ can be extracted.

J_q computed by Lattice QCD w/ big m_q^{latt} .
 \rightarrow \propto corrections

(Mathur, Dong, Liu, Mankiewicz, Mukhopadhyay '00;
Gadgil, Ji, Jung '01)

Chiral logs of J_8

$$J_8 = \underbrace{\text{---} \blacksquare \text{---}}_{L_0} + \underbrace{\text{---} \blacksquare \text{---} + \text{---} \blacksquare \text{---}}_{NLO} + \dots$$

$$= J_8^{(0)} - \left(J_8^{(0)} - \frac{\langle X \rangle_{8/\pi}}{2} \right) \frac{3g_A^2}{(4\pi f_\pi)^2} m_\pi^2 \ln m_\pi^2 + \dots$$

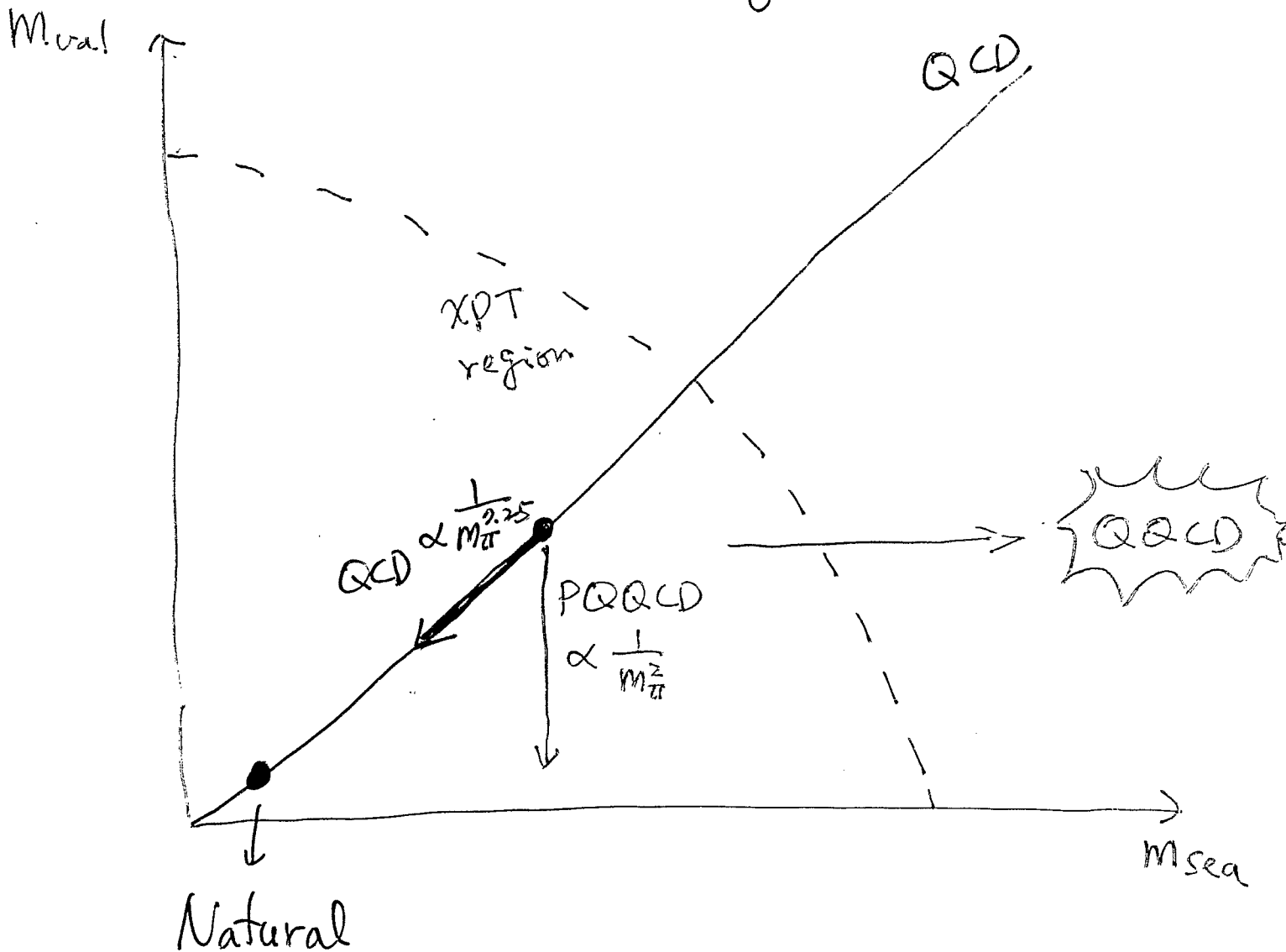
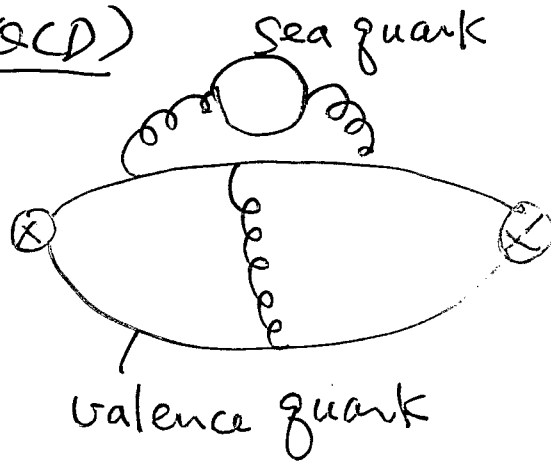
If $J_8^{(0)} = \frac{\langle X \rangle_{8/\pi}}{2} \sim 0.25$, $m_\pi^2 \ln m_\pi^2$ term largely canceled, favored scenario.

\Rightarrow linear extrapolation in m_π^2 might be robust

Quenched and Partially Quenched

QCD (QQCD & PQQCD)

$\langle 0 | \pi \pi | 0 \rangle$



QCD:

$$\begin{aligned} \langle X^n \rangle_{u-d} &= \text{---} \bullet \text{---} + \text{---} \bullet \text{---} + \text{---} \bullet \text{---} \\ &= \langle X^n \rangle_{u-d}^0 \left(1 - \frac{3g_A^2 + 1}{(4\pi f_\pi)^2} m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \right) \end{aligned}$$

QQCD:

$$\begin{aligned} \langle X^n \rangle_{u-d} &= \text{---} \text{P} \bullet \text{---} + \text{---} \text{P} \bullet \text{---} \text{N} \text{---} + \text{---} \text{P} \bullet \text{---} \\ &\quad \text{"K"} = \overline{\text{ghost } g} \\ &\quad + \text{---} \text{---} \bullet \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \bullet \text{---} \text{---} \text{---} + \dots \\ &\quad \text{"\Sigma"} = \text{ghost } g g \quad \text{"\Lambda"} \end{aligned}$$

(no hairpins!)

$$= \frac{1}{3} (2\alpha^{(n)} + \beta^{(n)})$$

$$\times \left\{ 1 + \left[2D(D-3F) - \frac{2D(D-F)(\alpha^{(n)} + \beta^{(n)})}{\frac{1}{3}(2\alpha^{(n)} + \beta^{(n)})} \right] m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} \right\}$$

+ ...

(JWC + Savage)

$$\text{---} \text{---} \bullet \text{---} \text{---} \propto \text{---} \text{P} \bullet \text{---} (1 + \mathcal{O}(\frac{1}{N_c^2}))$$

$$\frac{F}{D} = \frac{2}{3} \left(1 + \mathcal{O}(\frac{1}{N_c^2}) \right)$$

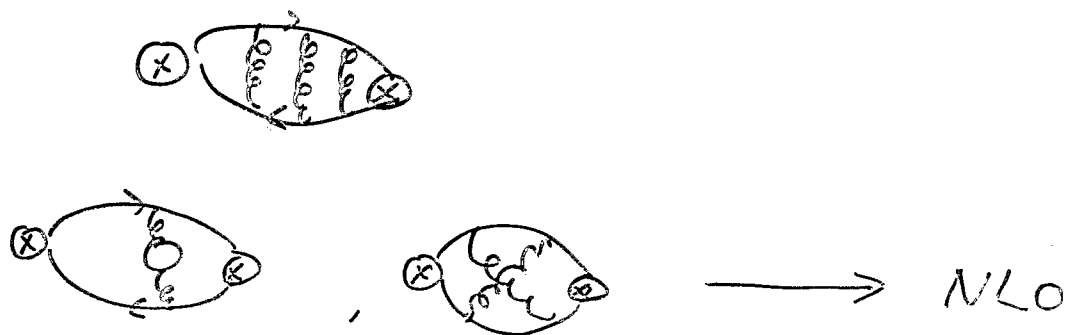
(Chen, in preparation)

∴ The large N_c world & the Quenched World

I. the large N_c world ($N_c \rightarrow \infty, g^2/N_c = \text{const}$)

Meson 2-pt function

LO: Coplanar diagrams



II the quenched world

$$\text{Diagram with ghost loop} = 0$$

However, $\text{Diagram with gluon loop} \rightarrow \text{LO}$

$$m_\pi^2 = m_\pi^{o2} \left[\left(1 + \frac{1}{N_c} \log s\right) + C_1 m_{g,\text{val}} + \frac{C_2}{N_c} m_{g,\text{sea}} \right]$$

$$\log s = \frac{m_\pi^2 \ln m_\pi^2}{(4\pi f_\pi)^2}, \quad \frac{m_\pi^2 \ln m_\pi^2}{(4\pi f_\pi)^2}, \quad f_\pi \sim \sqrt{N_c}, \quad m_0 \sim \sqrt{N_c}$$

* $\log s$ are big as $m_\pi \rightarrow 0$

* m_π^{o2} & C_1 at $N_c \rightarrow \infty$ are the same in QCD & QQCD

$$\Rightarrow (m_\pi^2, C_1)_{\text{quenched}} = \frac{1}{64} (m_\pi, C_1)_{\text{QCD}} (1 + \mathcal{O}(1/N_c))$$

CONNECTING STRUCTURE FUNCTIONS ON THE LATTICE WITH PHENOMENOLOGY

W. DETMOLD¹, W. MELNITCHOUK², A. W. THOMAS¹

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and Mathematical Physics, Adelaide University 5005, Australia*

² *Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

In this report we have highlighted the importance of model independent constraints from the chiral and heavy quark limits of QCD in the extrapolation of lattice data on parton distribution moments. Inclusion of the nonanalytic structure associated with the infrared behavior of Goldstone boson loops leads to a resolution of a long-standing discrepancy between lattice data on low moments of the spin-averaged $u - d$ distribution and experiment.

The importance of ensuring the correct chiral behavior is further illustrated by comparing the x distributions obtained by extrapolating the lattice data using a linear and a chirally symmetric fit. While the latter gives an x distribution which is in quite good agreement with the phenomenological fits, the linearly extrapolated data give distributions with the wrong small- x behavior, which translates into a much more pronounced peak at $x \sim 1/3$, reminiscent of a heavy, constituent quark-like distribution. Our analysis suggests an intriguing connection between the small- x behavior of the valence distributions and the m_q dependence of meson masses on Regge trajectories, which should be tested more thoroughly in future simulations of the excited hadron spectrum.

Finally, we have highlighted the need for further study of moments of the helicity distributions, and the axial vector charge of the nucleon in particular, which appears to be underestimated in lattice simulations. The inclusion of the pion cloud of the nucleon leads to a larger discrepancy at the physical quark mass, indicating that lattice artifacts such as finite volume effects may not yet be under control. Further data on larger lattices, and at smaller quark masses, will be necessary to resolve this issue.

References

1. W. Detmold, W. Melnitchouk, J. W. Negele, D. B. Renner and A. W. Thomas. *Phys. Rev. Lett.* **87**, 172001 (2001).
2. A.W. Thomas, W. Melnitchouk and F.M. Steffens, *Phys. Rev. Lett.* **85**, 2892 (2000).
3. W. Detmold, W. Melnitchouk and A.W. Thomas, *Eur. Phys. J. direct C* **13**, 1 (2001).

Structure Functions in Lattice QCD

Cannot calculate x -distribution, only moments

$$\langle x^n \rangle = \int_0^1 dx \, x^n \left(q(x) + (-1)^{n+1} \bar{q}(x) \right)$$

where $\langle x^n \rangle$ are matrix elements of twist-2 operators (twist = dimension – spin):

$$\langle x^n \rangle \, p_{\mu_1} \cdots p_{\mu_{n+1}} = \langle N | \mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} | N \rangle$$

where

$$\mathcal{O}_{\{\mu_1 \cdots \mu_{n+1}\}} = \bar{\psi} \, \gamma_{\{\mu_1} \, \overleftrightarrow{D}_{\mu_2} \cdots \overleftrightarrow{D}_{\mu_{n+1}} \} \, \psi$$

On the lattice, operators transform according to hypercubic $H(4)$ subgroup of $O(4)$

→ because of operator mixing, only $n \leq 3$ moments can be calculated

Chiral Symmetry and QCD

- In limit $m_q \rightarrow 0$, QCD has exact *chiral* $SU(N_f)_L \times SU(N_f)_R$ symmetry
- Spontaneous breaking of chiral symmetry
→ appearance of pseudoscalar Goldstone bosons (π, K)
- Pions play important role at small $m_\pi^2 \sim m_q$
→ expand observables in powers of m_π
(e.g. χPT)
- Pion loops in χPT generate non-polynomial terms which are *non-analytic* in m_q
(odd powers of m_π or $\log m_\pi$)
- Coefficients of non-analytic terms are model-independent
→ given in terms of g_A and f_π

- Lattice calculations employ $m_q > 30 \text{ MeV}$
 → need to extrapolate to physical m_q
- m_q dependence of observables
 → constrained by χPT in $m_q \rightarrow 0$ limit
 Note: expansion may not converge at large m_π !
 → constrained by heavy quark effective theory in $m_q \rightarrow \infty$ limit
- Physics picture — source of π field is a complex system of quarks and gluons with finite size, Λ
 - when $m_\pi > \Lambda$, π loops suppressed, hadron observables smooth in m_q (“constituent quark-like” behavior)
 - when $m_\pi < \Lambda$, rapid non-linear variation
 - transition occurs at $m_\pi \sim 500 \text{ MeV}$

Chiral Extrapolation of SF Moments

Extrapolation formula for n -th moment of $u-d$:

$$\begin{aligned}\langle x^n \rangle_{u-d} = & a_n + b_n \frac{m_\pi^2}{m_\pi^2 + m_b^2} \\ & + c_n m_\pi^2 \log \left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right)\end{aligned}$$

Detmold, WM, Negele, Renner, Thomas,
Phys.Rev.Lett. **87** (2001) 172001
hep-lat/0103006

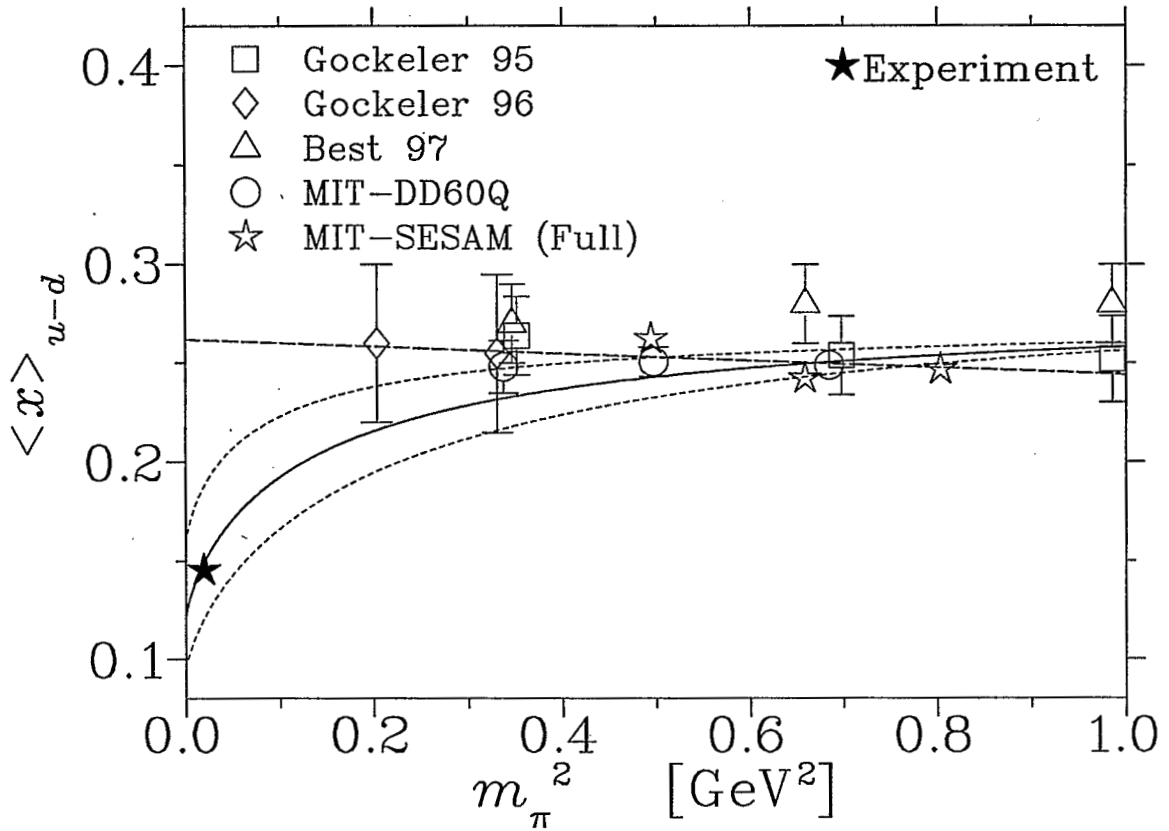
Thomas, WM, Steffens,
Phys.Rev.Lett. **85** (2009) 2892
hep-lat/0103006

Coefficient of non-analytic term:

$$c_n = -a_n \frac{3g_A^2 + 1}{4\pi^2 f_\pi^2}$$

calculated in chiral perturbation theory

Arndt, Savage, nucl-th/0105045
Chen, Ji, hep-ph/0105197



$n = 1$ moment of $u - d$ distribution:
best fit to data using chirally-improved extrapolation,
with $\mu = 550$ MeV (solid), and $\mu = 400$ and 700 MeV
(upper and lower dotted), compared with a linear
extrapolation (dashed).

Detmold, WM, Thomas
hep-lat/0201288

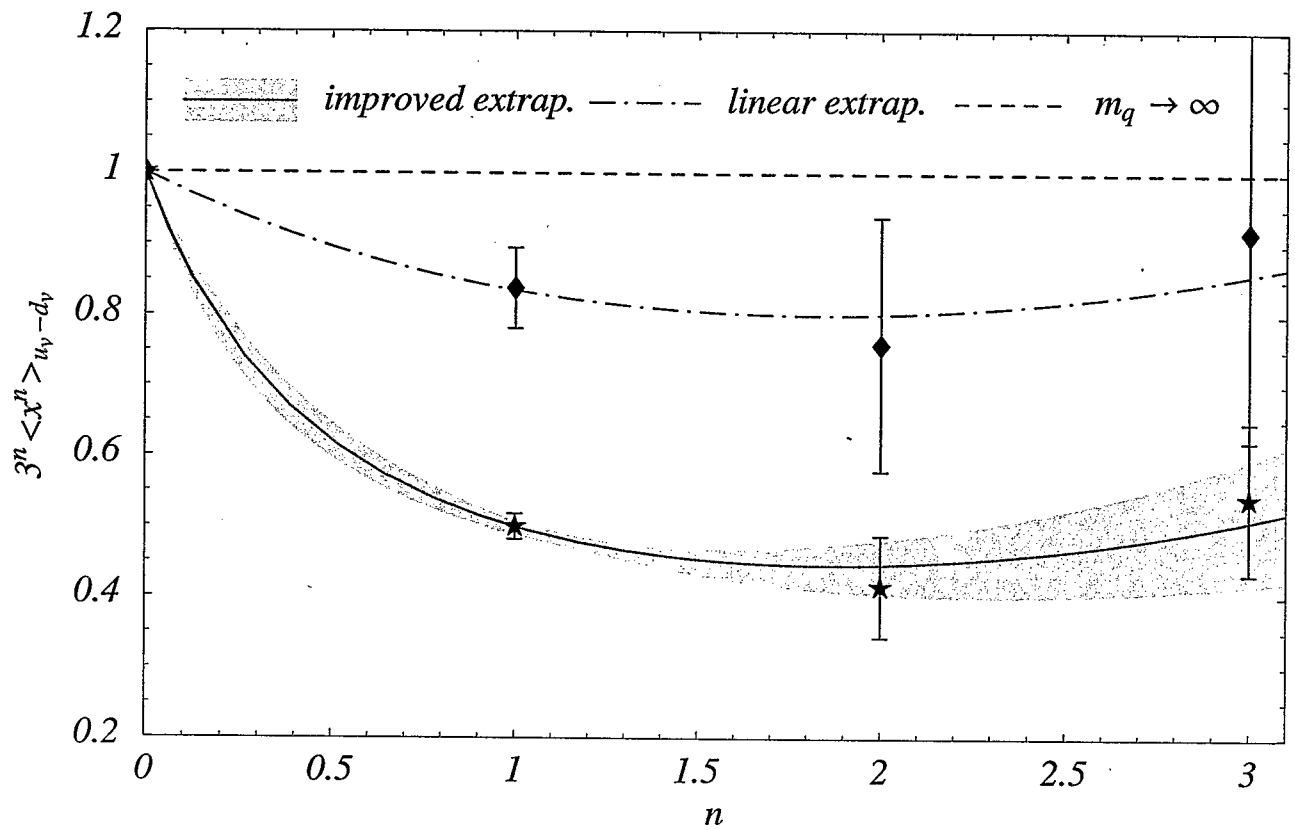
Detmold, WM, Negele, Renner, Thomas,
Phys.Rev.Lett. **87** (2001) 172001
hep-lat/0103006

x dependence and Regge Trajectories

- What can lattice moments tell us about x dependence of parton distributions?
- Typical parameterization of (non-singlet) distributions (CTEQ, MRS, GRV)

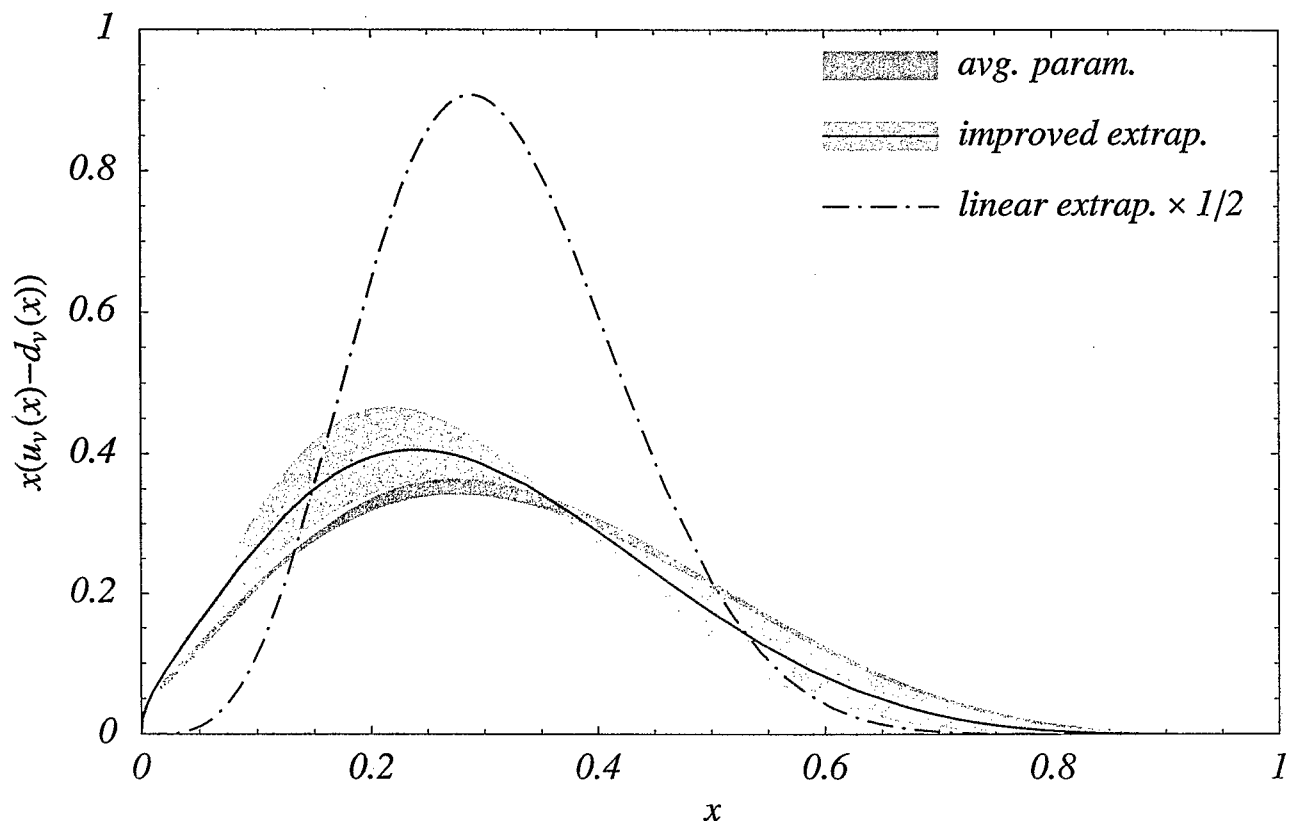
$$x q(x) = a x^b (1-x)^c (1 + \epsilon\sqrt{x} + \gamma x)$$

- Fit parameters b and c to lattice data using extrapolation formula
→ quark distribution as function of m_q !
- Exponent b related to intercept of a_2 Regge trajectory
→ relate m_q dependence of masses of $1^{--}, 2^{++}, 3^{--}, \dots$ mesons to $x \rightarrow 0$ behavior of structure function moments!



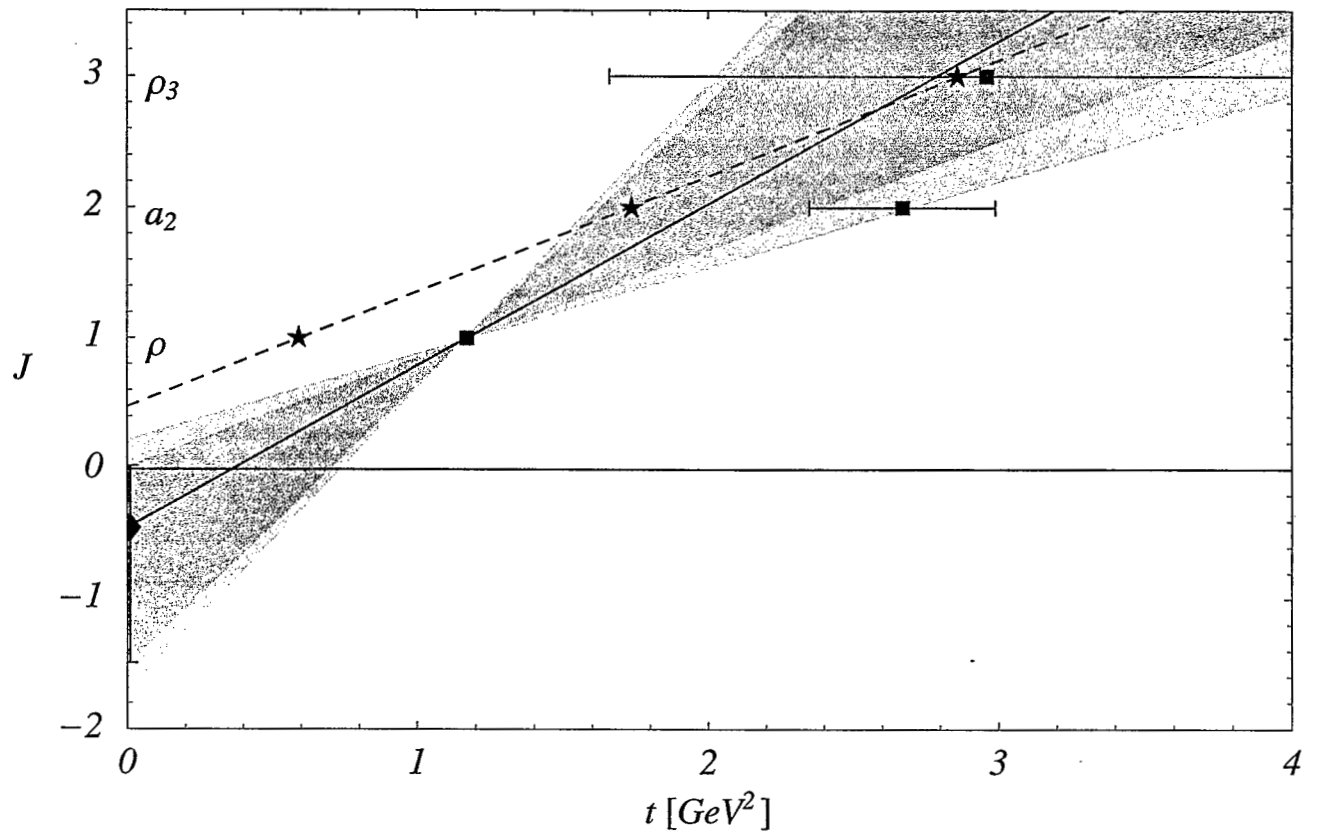
Moments of $u_v - d_v$ distribution
extracted from lattice data (scaled by 3^n)

Detmold, WM, Thomas
Eur.Phys.J. C 13 (2001) 1
hep-lat/0108002



Extracted $x(u_v - d_v)$ distribution
at the physical quark mass

Detmold, WM, Thomas
Eur.Phys.J. C 13 (2001) 1
hep-lat/0108002



Regge plot of mesons on the
 a_2 trajectory

Lattice data from UKQCD (at $m_q = m_s$)

Detmold, WM, Thomas
Eur.Phys.J. C 13 (2001) 1
 hep-lat/0108002

Chiral Symmetry with Overlap Fermions

by

Keh-Fei Liu

Dept. of Physics and Astronomy
University of Kentucky
Lexington, KY 40506

After a synopsis on overlap fermions I shall present some results to show that the quark mass does not have additive renormalization and the $O(a^2)$ error is small both in hadron masses and in chiral Ward identity. The quenched chiral logs and the hadron masses are studied on 20^4 lattice with $a = 0.148$ fm. I also use local chirality and topological density of the low-lying Dirac eigenmodes to study chiral symmetry breaking and the scenario of the instanton liquid model.

Chiral Symmetry with Overlap Fermions *

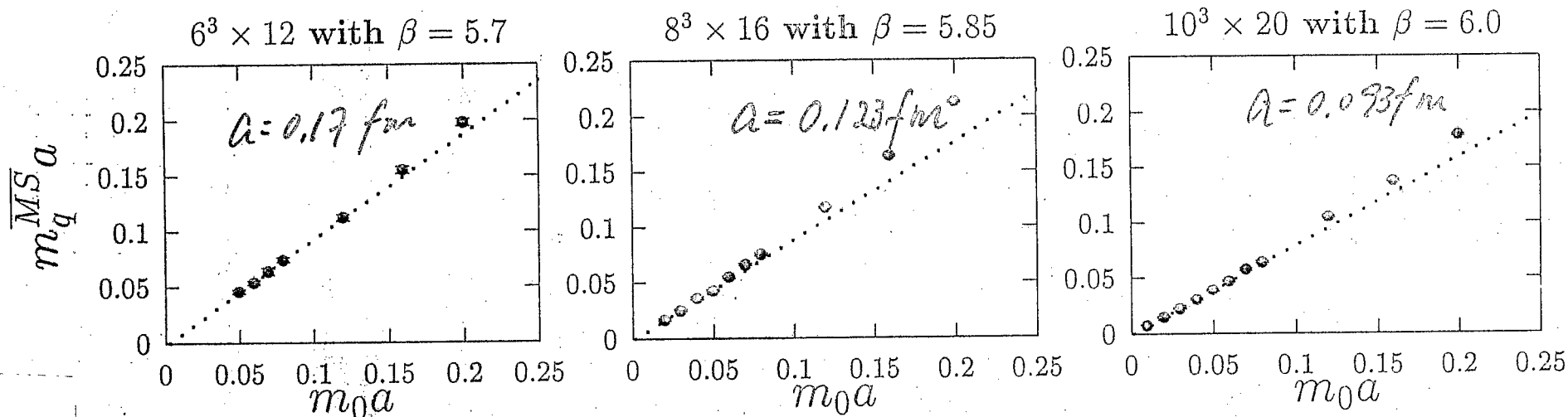
- Synopsis of Overlap Fermions
- Axial Ward Identity, Quark Mass, Scaling, Z_A , and f_π
- Quenched Chiral Logs
- Hadron Masses, Roper Resonance
- Chiral Symmetry Breaking Scenario via Instantons
- Local Chirality
- Local Topology from Fermion Eigenmodes

Collaborators:

- Hadron Properties
S.J. Dong, T. Draper, I. Horvath, F.X. Lee, and J.B. Zhang
- Local Chirality and Local Topology
S.J. Dong, T. Draper, I. Horvath, N. Isgur, F.X. Lee, J. McCune, H.B. Thacker,
and J.B. Zhang

*Talk presented by Keh-Fei Liu at Brookhaven Workshop on Hadron Structure from Lattice QCD. Mar. 19, 2002

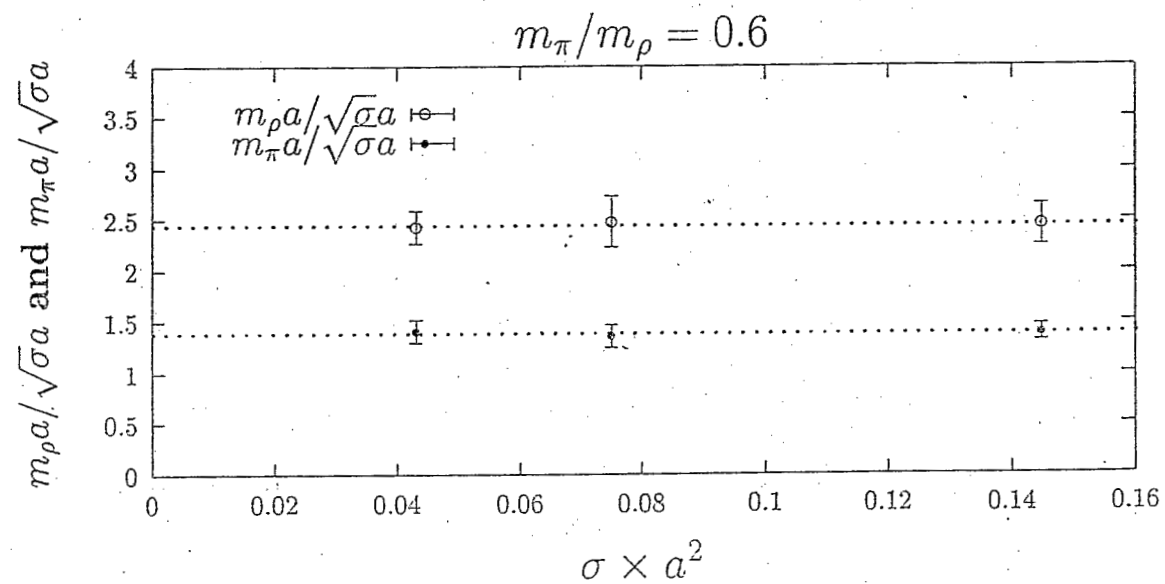
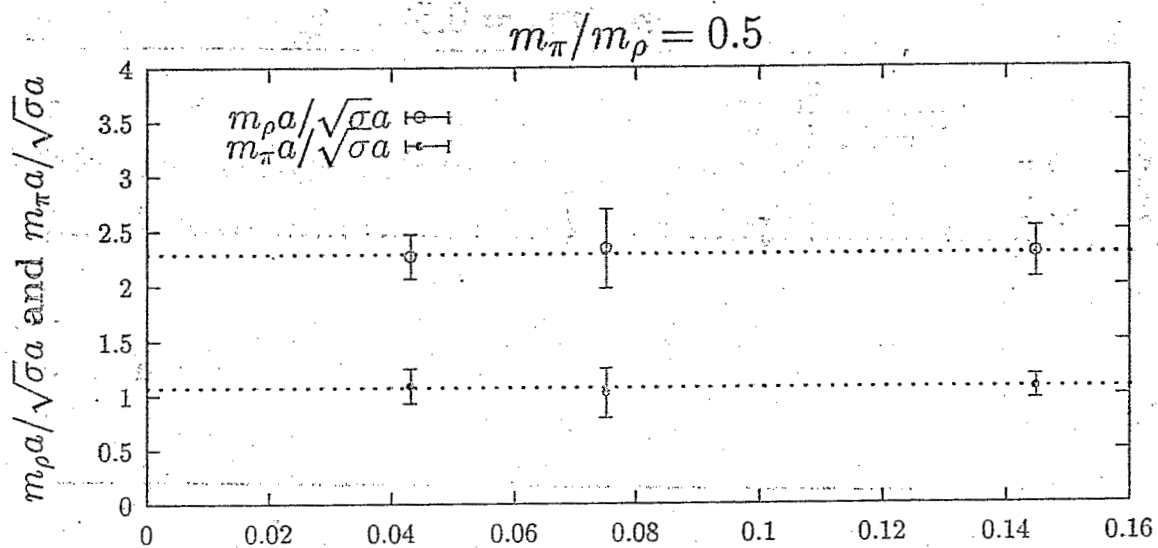
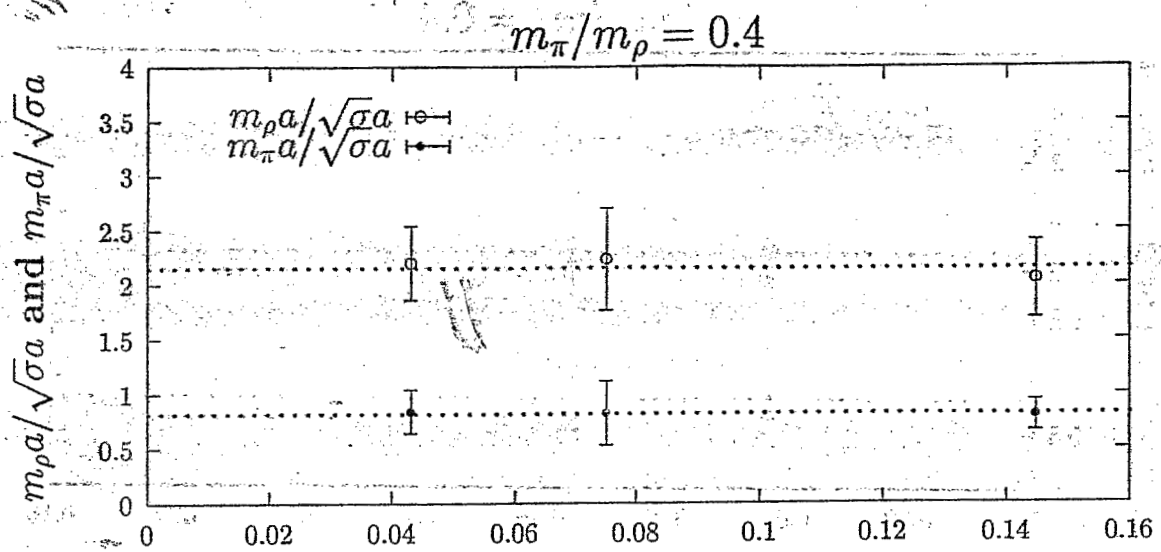
Renormalized Quark Mass



Axial Ward Identity : $Z_A \partial_\mu A_\mu = 2 Z_S^{-1} m_0 Z_P P$

$$a m_f^{\overline{MS}}(\mu) = \lim_{t \rightarrow \infty} \frac{Z_A(\mu)}{Z_P(\mu)} \frac{\sum_x \langle 0 | \nabla_\mu A_\mu(x) | \pi(0) \rangle}{2 \sum_x \langle 0 | P(x) | \pi(0) \rangle}$$

Scaling



Z_A and f_π

Z_A is determined to the precision of a fraction of 1%.

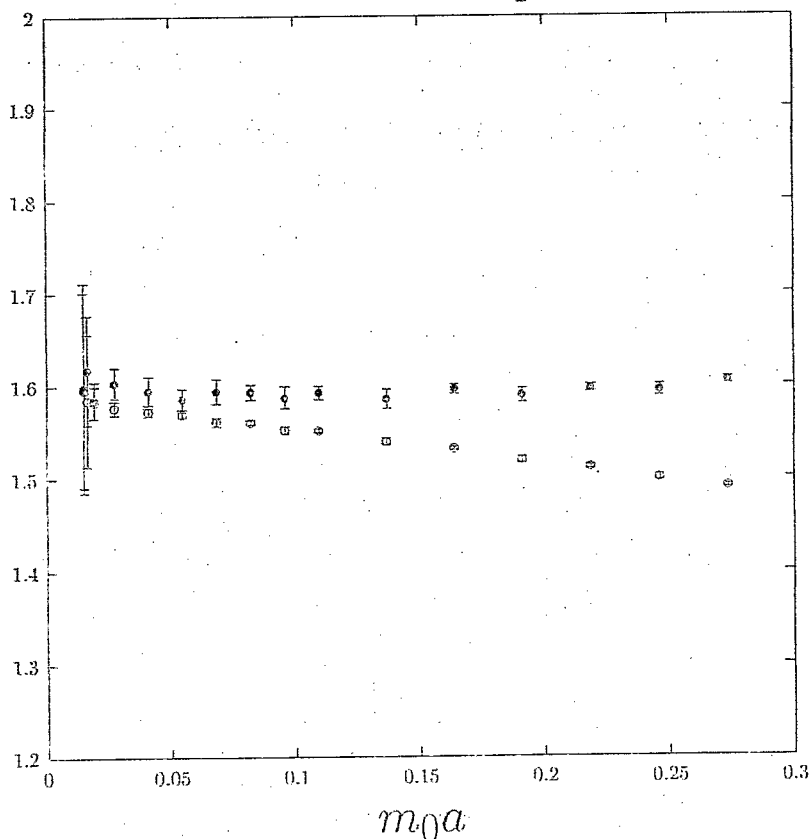
From the axial Ward identity

$$Z_A \partial_\mu A_\mu = 2Z_m m_0 Z_P P$$

and the relations $Z_m = Z_S^{-1}$ and $Z_S = Z_P$ valid for overlap fermions, one obtains the non-perturbative renormalization constant Z_A

$$\begin{aligned} Z_A &= \lim_{t \rightarrow \infty} \frac{2m_0 G_{PP}(\vec{p}=0, t)}{G_{\partial_4 A_4 P}(\vec{p}=0, t)} \\ &= \lim_{t \rightarrow \infty} \frac{2m_0 G_{PP}(\vec{p}=0, t)}{m_\pi G_{A_4 P}(\vec{p}=0, t)} \end{aligned}$$

20⁴ Lattice with Overlap Fermions



Z_A vs quark mass $m_0 a$. The discretized partial derivative induces a discretization error (for \circ). Z_A (\bullet) is conspicuously flat as a function of $m_0 a$ indicating that the $O(a^2)$ error from the action and the operators is small.

Fit to the form $Z_A + b m_0^2 a^2$: $Z_A = 1.589(4)$, $b = 0.175(73)$, $\chi^2/DF = 0.30$.

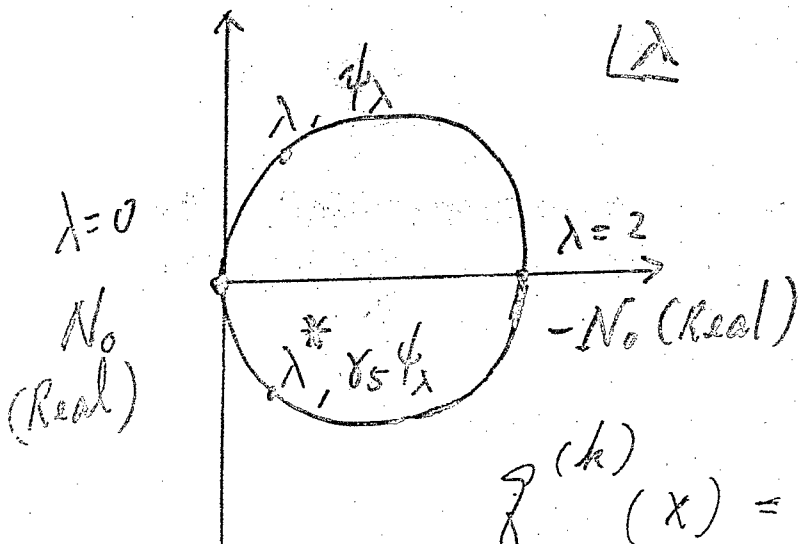
{ Local Topology Hasenfratz,
Laliena,
Niedermayer

$$g(x) = \frac{1}{2} \text{Tr} (\gamma_5 D(x, x))$$

$$= - \text{Tr} (\gamma_5 (1 - \frac{1}{2} D(x, x)))$$

$$= - \sum_{\lambda} (1 - \frac{\lambda}{2}) \psi_{\lambda}^{\dagger}(x) \gamma_5 \psi_{\lambda}(x)$$

↗ local chirality



$$g^{(k)}(x) = - \sum_{i=1}^{N_0} \psi_0^{\dagger} \gamma_5 \psi_0(x)$$

$$- \sum_{i=1}^k (1 - \text{Re} \lambda_i) \psi_{\lambda_i}^{\dagger} \gamma_5 \psi_{\lambda_i}(x)$$

when $\text{Re} \lambda_i \ll 1$, $g^{(k)}(x) \sim \sum_{i=1}^k \psi_{\lambda_i}^{\dagger} \gamma_5 \psi_{\lambda_i}(x)$

↗ local chira.

Excited Baryon Spectroscopy

David Richards (JLab and LHP Collaboration)

Jefferson Lab.

- Baryons in Lattice QCD
- Negative-parity baryons and N^* 's
- Mysteries - Roper resonance and $\Lambda(1405)^-$
- Anisotropic action - *preliminary*
- Discussion

Work with M. Göckeler *et al.*, hep-lat/0106022, PLB
(to appear)

Material from Wally Melnitchouk (CSSM and JLab)

Excited Baryon Spectroscopy

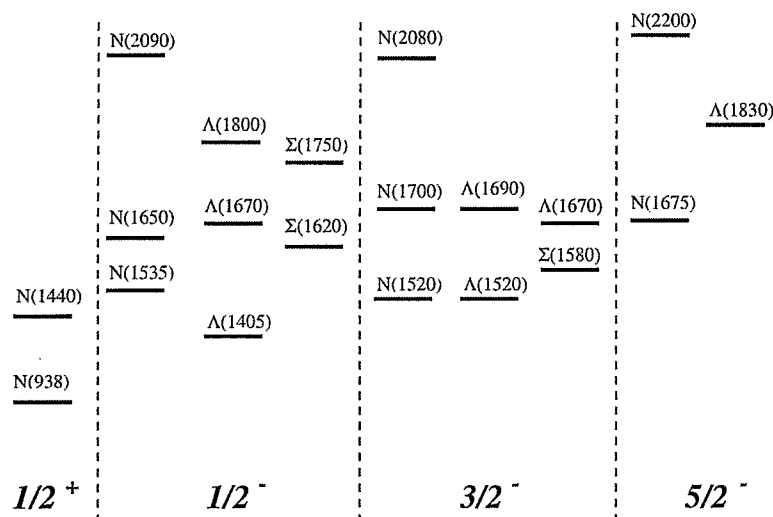
The N^* spectrum exhibits many features that are emblematic of QCD; that they contain three valence quarks is a manifestation of $SU(3)$ symmetry, while the linearly rising Regge trajectories are evidence for a flux-tube-like confining force. Finally, the fine and hyperfine structure reveals details of the interquark interactions. Thus the study of the spectrum of excited nucleon resonances has always been a vital component of the experimental hadronic physics program, as typified by the Hall B experimental effort at Jefferson Laboratory. In many respects the study of the N^* resonances mirrors analogous investigations of the hybrid meson spectrum, with the benefit that the extra degrees of freedom in the baryon sector admit a far richer spectrum than for mesons.

The observed excited nucleon spectrum already poses many questions. There are “missing resonances”, predicted by the quark model but not yet observed experimentally. Most strikingly, there is the so-called Roper resonance, and the Λ parity partner, the $\Lambda(1405)$, with anomalously light masses suggesting that they may be “molecular” states, rather than true three-quark resonances. Lattice gauge theory calculations have a vital role to play in resolving these questions.

There have been several recent calculations of the lowest-lying excited nucleon masses, emphasising in particular the extent to which the parity partners are accessible to lattice calculation. The LHPC/QCDSF/UKQCD calculation (hep-lat/0106022, to appear in Phys. Letter **B**) using the $\mathcal{O}(a)$ -improved clover fermion action in the quenched approximation to QCD finds a splitting between the nucleon and its parity partner $N^*(1535)$ consistent with experiment, after extrapolation to the continuum limit. All lattice calculations find a mass for the first radial excitation of the nucleon far higher than the Roper resonance, with a splitting between the radial excitation of the nucleon and the $N^*(1535)$ comparable to that between the parity partner and the nucleon. Recently, the CSSM has also shown that there is little evidence for a $\Lambda(1405)^-$ as a three-quark state (hep-lat/0202022). Thus lattice calculations are already posing important phenomenological questions.

A more complete study of the spectrum will require the full panoply of lattice technology: variational methods, anisotropic lattices and other noise-reduction techniques, and the construction of better interpolating operators. A preliminary study using the anisotropic clover action suggests that such a technique can provide a important improvement in the signal-to-noise ratio.

Motivation



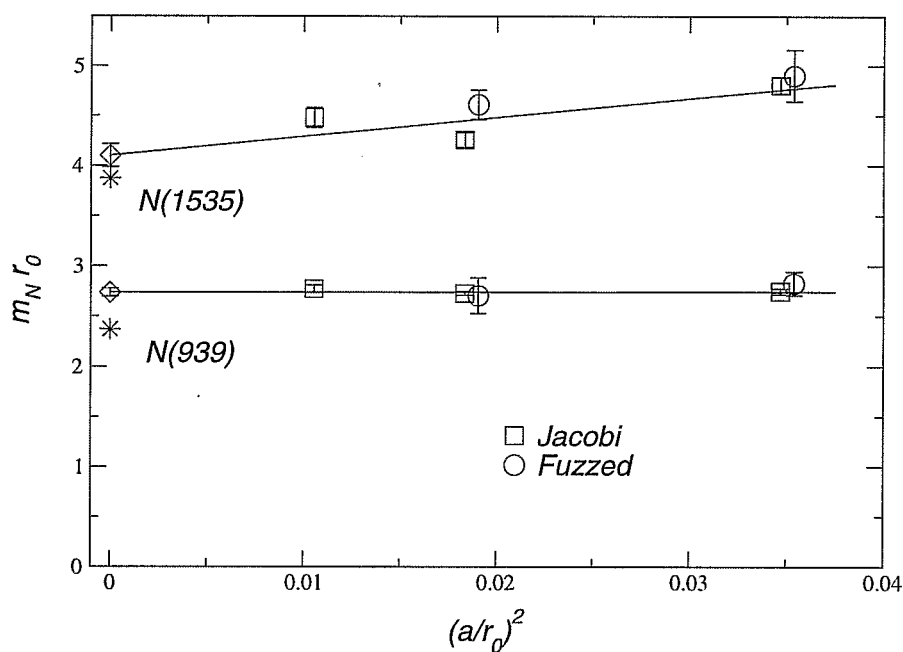
- Describing N^* spectrum gives vital clues about dynamics of QCD and hadronic physics
 - Role of excited glue
 - Quark-diquark picture
 - Quark interactions
- c.f.* Hybrids - except all J^P states accessible to quark model.
- N^* spectrum important element of JLAB Physics.
- Open mysteries:
 - Nature of *Roper*?
 - $\Lambda(1405)$?
 - Missing resonances?

Nucleon and Parity Partner

Continuum Extrapolations

M. Göckeler *et al*, hep-lat/0106022

Express masses in units of r_0 :



r_0 poor scale for comparing with experiment

$$M_{N^{1/2-}}/M_{N^{1/2+}} = 1.50(3)$$

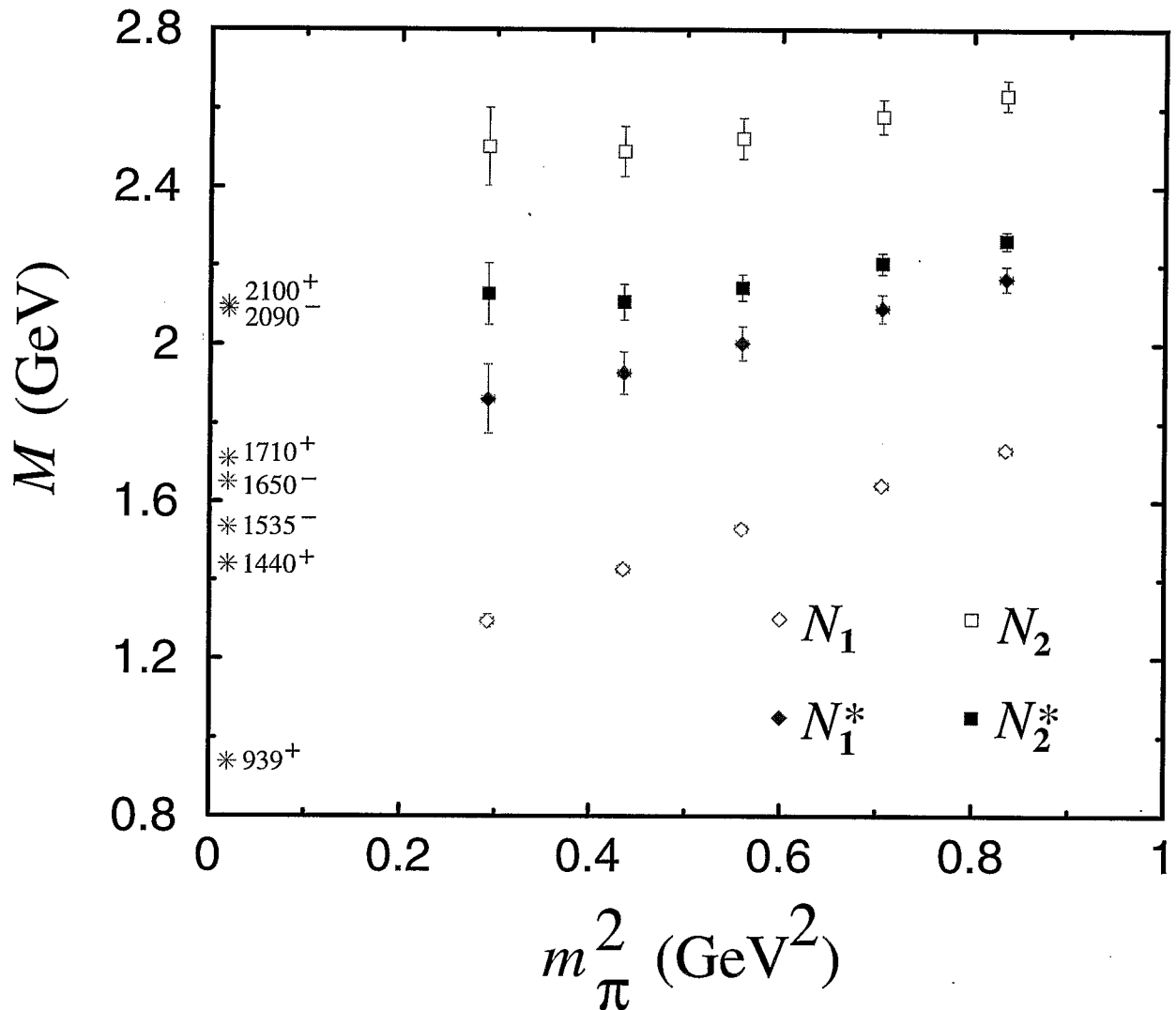
5% finite-size

10% discretisation

$N^*(1/2^\pm)$ states

CSSM Adelaide Lattice Collaboration

Melnitchouk et al. hep-lat/0202022

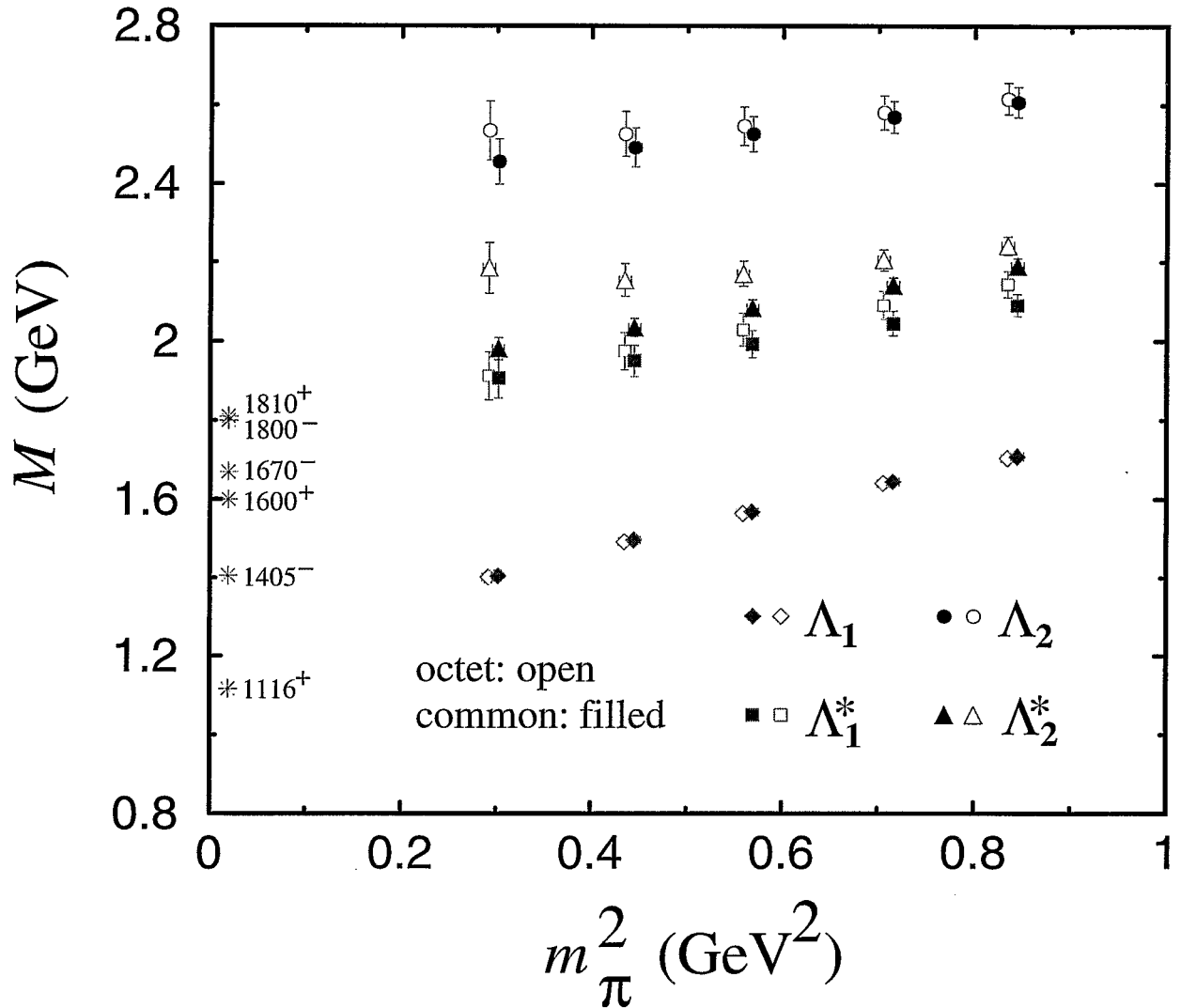


- No evidence of $N^*(1440)$ (Roper) state
- Evidence of mass splitting between the $N_1^*(1/2^-)$ and $N_2^*(1/2^-)$ fields
(c.f. $N^*(1535)$ and $N^*(1650)$ mass splitting)

$\Lambda^*(1/2^\pm)$ masses

CSSM Adelaide Lattice Collaboration

Melnitchouk et al. hep-lat/0202022

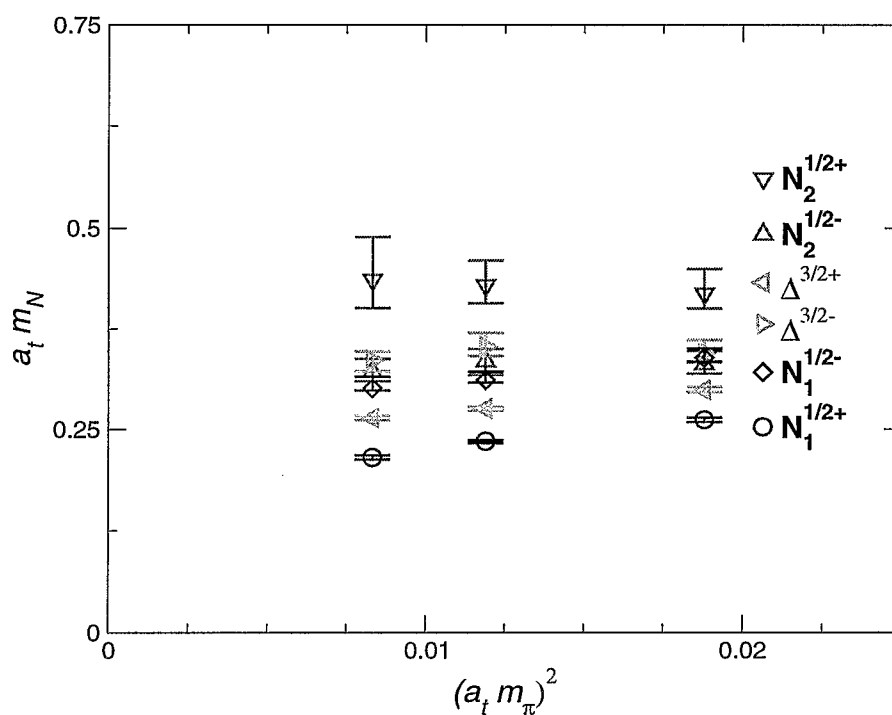


- No evidence of overlap with $\Lambda^*(1405)$
 - strong nonlinearity as $m_q \rightarrow 0$
 - more exotic interpolating fields required?

Anisotropic Clover Lattices

LHPC, preliminary

Use *finer* temporal lattice spacing to extract signal over more time slices



- $a_s^{-1} \sim 2$ GeV
- $a_t^{-1} \sim 6$ GeV
- $L_s \sim 2.4$ fm

Study of Nucleon Light Cone Quark Distributions in Quenched, Full, and Cooled Lattice QCD

John W. Negele

BNL Hadron Structure Workshop March 20, 2002

Outline

Introduction and motivation

Aspects of deep inelastic scattering

Calculation of moments of structure functions on the lattice

- Operators

- Renormalization

- Matrix elements

- Source optimization

- Consistency Checks

Results

- Quenched

- Full QCD

- Cooled configurations and instantons

- Chiral extrapolation

Summary and conclusions

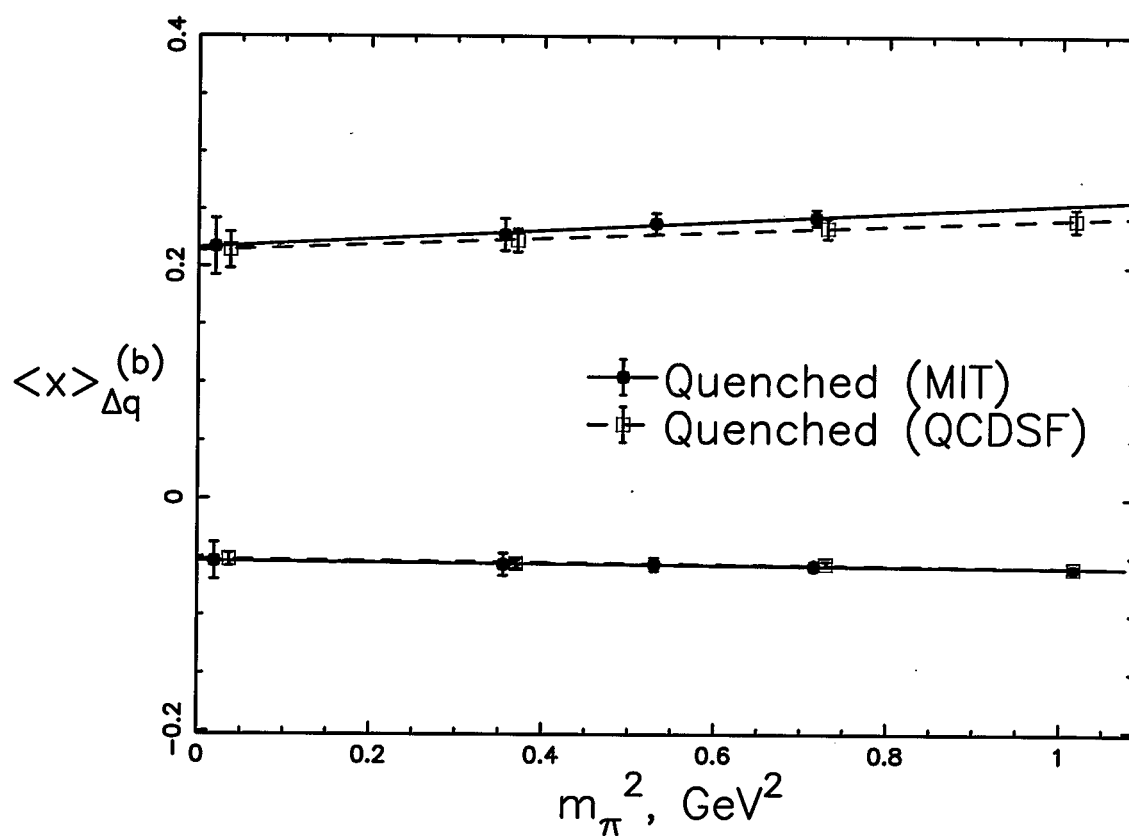
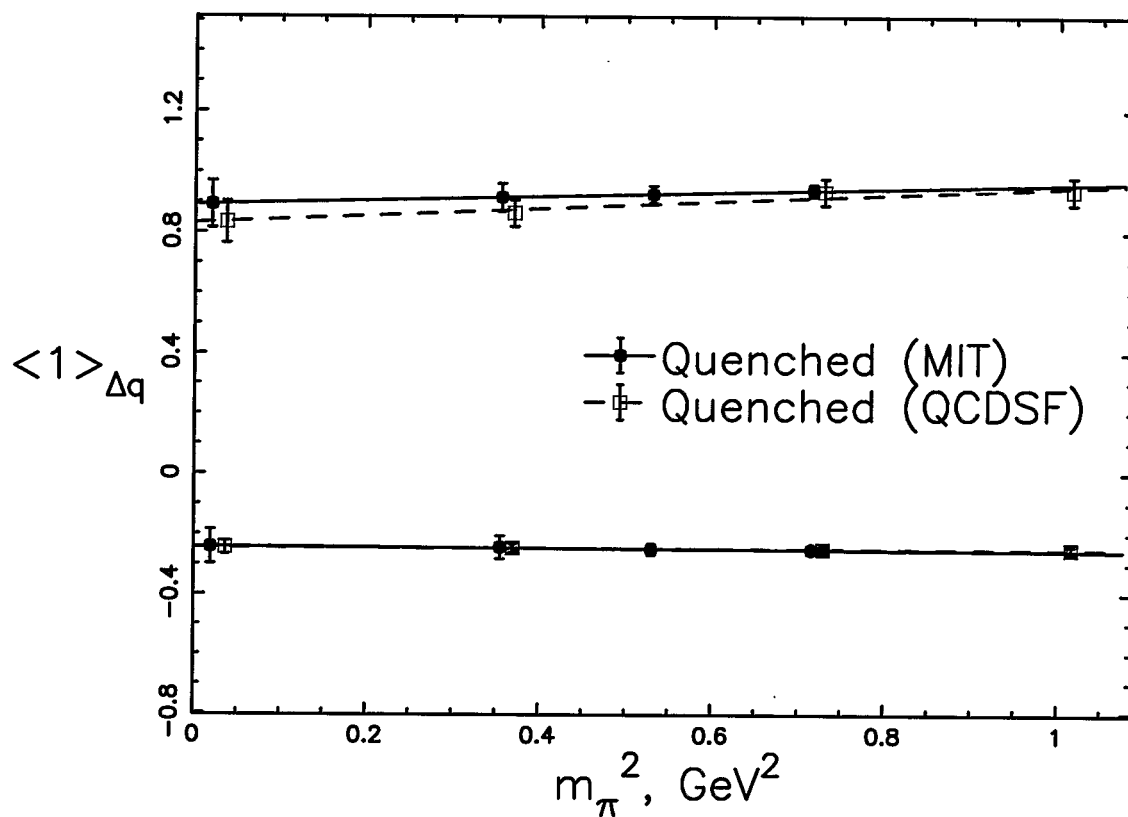
Summary

- First calculation of moments of quark distributions in full QCD
- Close agreement between full QCD and quenched
- Linear extrapolation in m_π^2
⇒ serious disagreement with experiment
- Evidence that proper extrapolation with chiral logs
will yield consistent results for all moments
- Qualitative agreement between full QCD and
cooled gluons at small m_q
⇒ instanton and zero-mode dominance

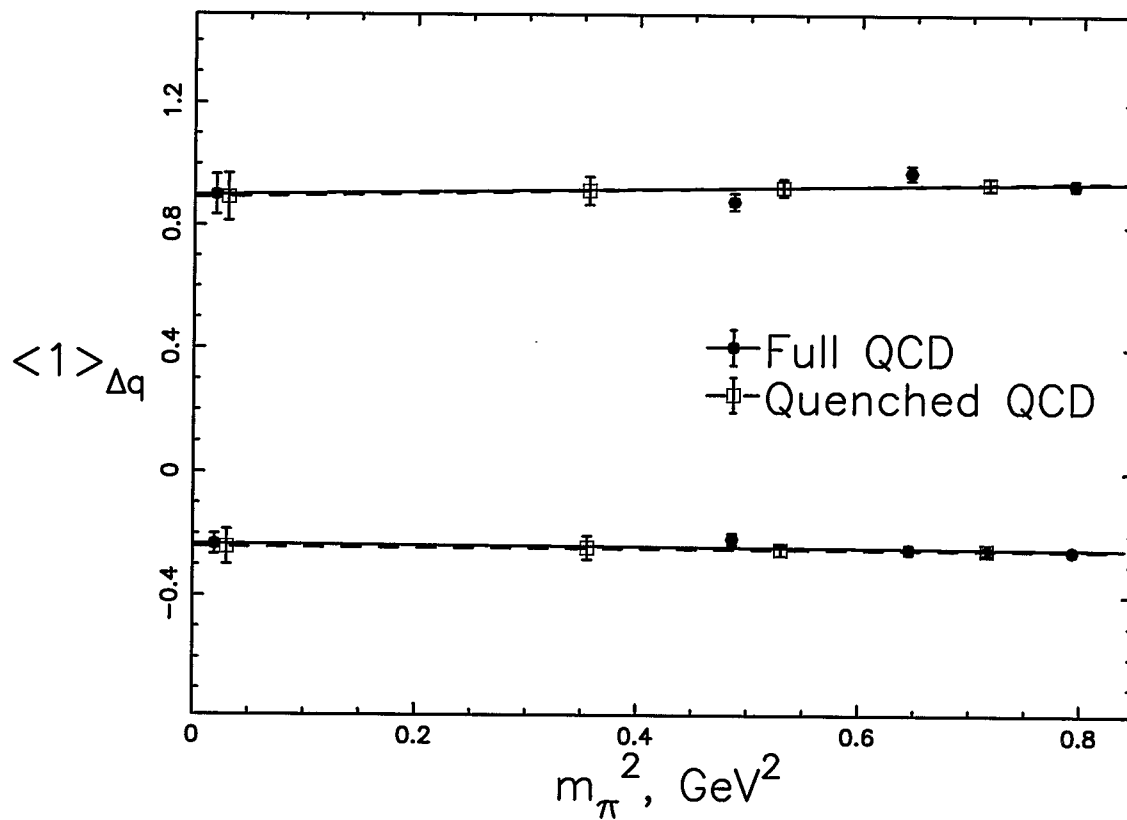
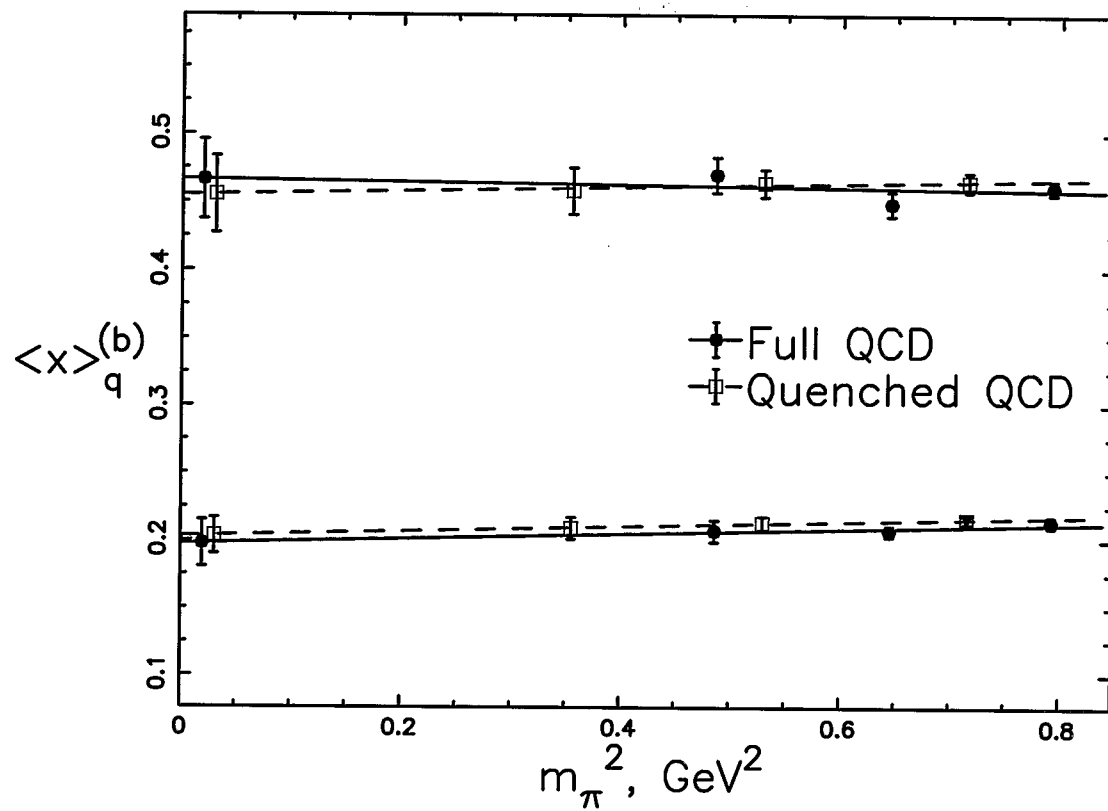
Future

- Systematic partially quenched chiral expansion
Measure parameters of effective chiral theory
Hybrid combination of staggered sea quarks and chiral valence quarks
- Disconnected diagrams
Exploit zero-mode dominance
- Chiral fermions
Improve renormalization
- Correction for fixed topology
- Finite volume corrections
- Gluon matrix elements
- Definitive Terascale calculations

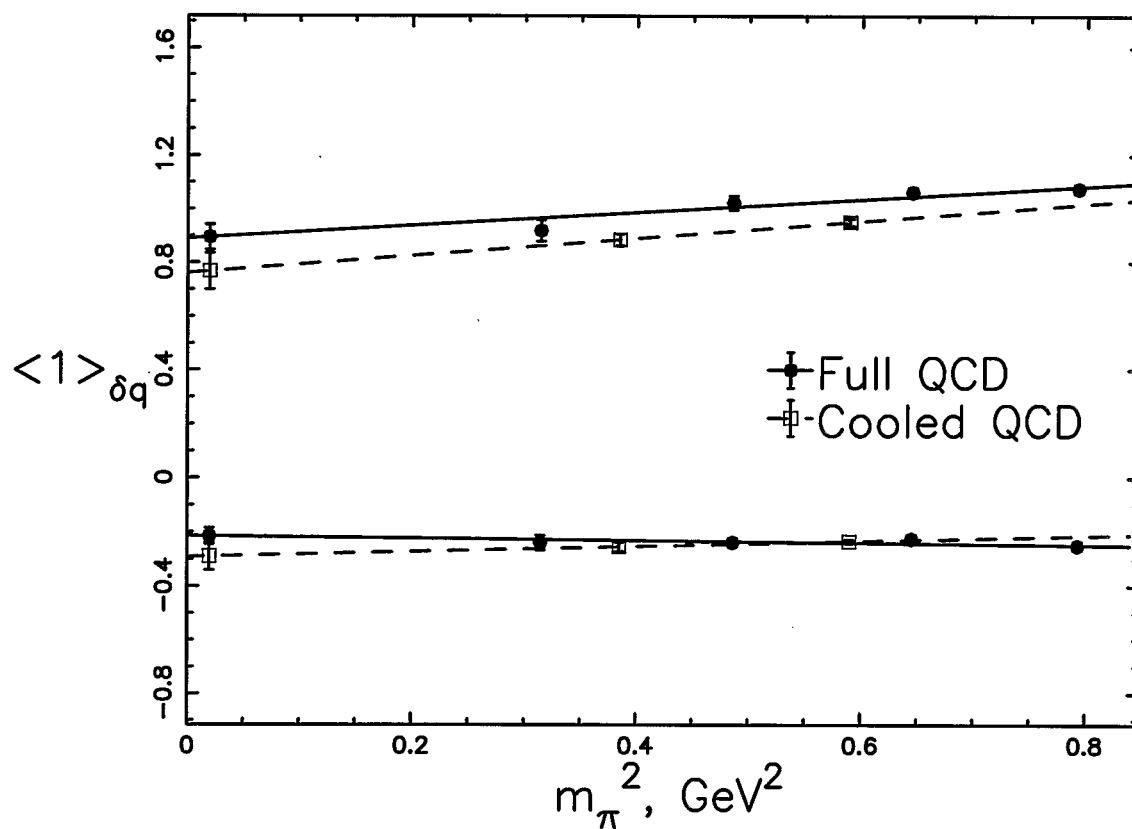
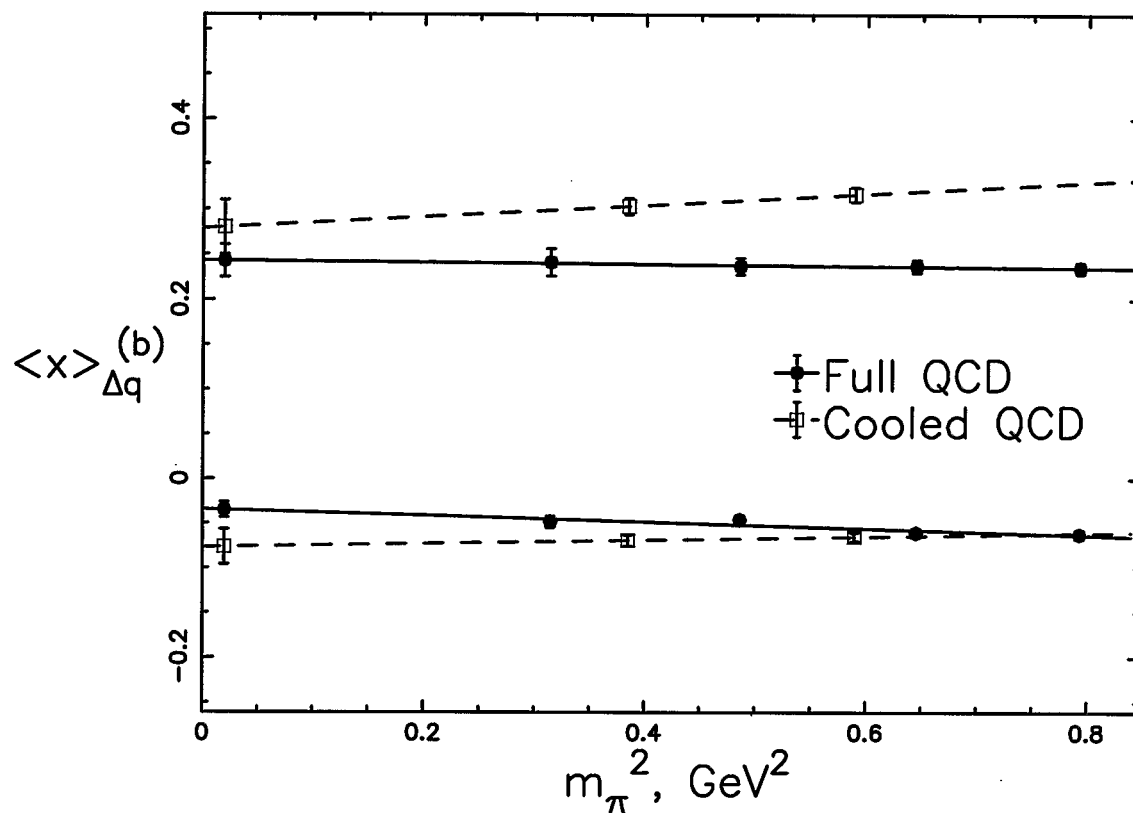
Comparison of Quenched Calculations (*cont.*)



Comparison of Full and Quenched QCD



Qualitative Behavior from Instantons (*cont.*)



Comparison with Other Calculations and Phenomenology

| Connected M. E. | QCDSF | QCDSF ($a = 0$) | Wuppertal | Quenched | Full QCD (3 pts) | Phenomenology ($q \pm \bar{q}$) |
|---|------------|----------------------|-----------|------------|---------------------|--------------------------------------|
| $\langle x \rangle_u$ | 0.452(26) | | | 0.454(29) | 0.459(29) | |
| $\langle x \rangle_d$ | 0.189(12) | | | 0.203(14) | 0.190(17) | |
| $\langle x \rangle_{u-d}$ | 0.263(17) | | | 0.251(18) | 0.269(23) | 0.154(3) |
| $\langle x^2 \rangle_u$ | 0.104(20) | | | 0.119(61) | 0.176(63) | |
| $\langle x^2 \rangle_d$ | 0.037(10) | | | 0.029(32) | 0.031(30) | |
| $\langle x^2 \rangle_{u-d}$ | 0.067(22) | | | 0.090(68) | 0.145(69) | 0.055(1) |
| $\langle x^3 \rangle_u$ | 0.022(11) | | | 0.037(36) | 0.069(39) | |
| $\langle x^3 \rangle_d$ | -0.001(7) | | | 0.009(18) | -0.010(15) | |
| $\langle x^3 \rangle_{u-d}$ | 0.023(13) | | | 0.028(49) | 0.078(41) | 0.023(1) |
| $\langle 1 \rangle_{\Delta u}$ | 0.830(70) | 0.889(29) | 0.816(20) | 0.888(80) | 0.860(69) | |
| $\langle 1 \rangle_{\Delta d}$ | -0.244(22) | -0.236(27) | -0.237(9) | -0.241(58) | -0.171(43) | |
| $\langle 1 \rangle_{\Delta u-\Delta d}$ | 1.074(90) | 1.14(3) | 1.053(27) | 1.129(98) | 1.031(81) | 1.248(2) |
| $\langle x \rangle_{\Delta u}$ | 0.198(8) | | | 0.215(25) | 0.242(22) | |
| $\langle x \rangle_{\Delta d}$ | -0.048(3) | | | -0.054(16) | -0.029(13) | |
| $\langle x \rangle_{\Delta u-\Delta d}$ | 0.246(9) | | | 0.269(29) | 0.271(25) | 0.196(9) |
| $\langle x^2 \rangle_{\Delta u}$ | 0.087(14) | | | 0.027(60) | 0.116(42) | |
| $\langle x^2 \rangle_{\Delta d}$ | -0.025(6) | | | -0.003(25) | 0.001(25) | |
| $\langle x^2 \rangle_{\Delta u-\Delta d}$ | 0.112(15) | | | 0.030(65) | 0.115(49) | 0.061(6) |
| δu_c | 0.93(3) | 0.980(30) | | 1.01(8) | 0.963(59) | |
| δd_c | -0.20(2) | -0.234(17) | | -0.20(5) | -0.202(36) | |
| d_2^u | -0.206(18) | | | -0.233(86) | -0.228(81) | |
| d_2^d | -0.035(6) | | | 0.040(31) | 0.077(31) | |

Physics Result – Chiral Extrapolation

- Long-standing puzzle: Linear extrapolation in m_q yields serious discrepancies

$$\langle x \rangle_u - \langle x \rangle_d \sim 0.24 - 0.28 \quad (0.16)$$

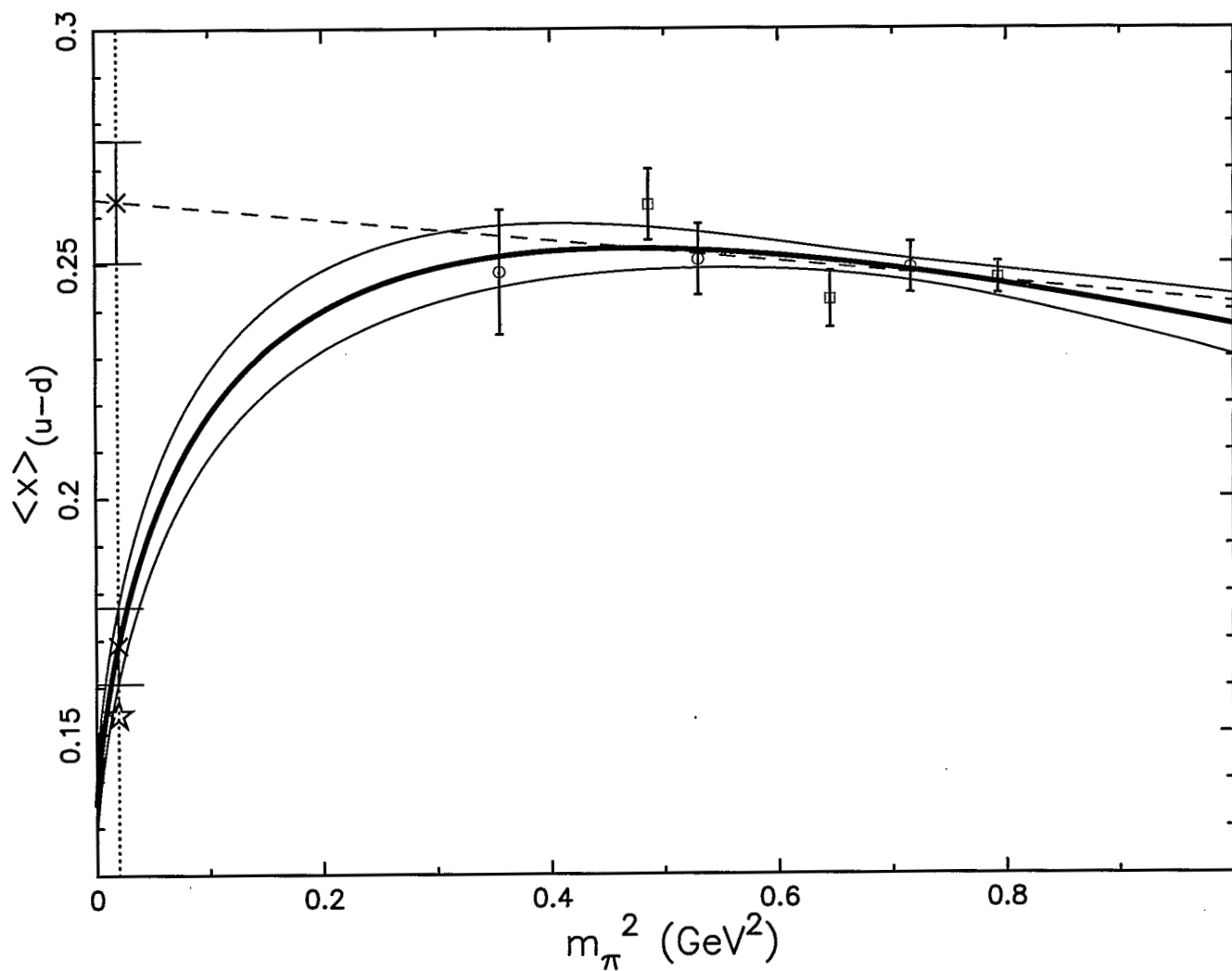
$$g_A = \langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d} \sim 1.0 - 1.1 \quad (1.26)$$

- Resolution: Chiral extrapolation

hep-lat/0103006

Pion cloud is essential

$$\langle x^n \rangle_u - \langle x^n \rangle_d \sim a_n \left[1 - \frac{(3g_A^2 + 1)m_\pi^2}{(4\pi f_\pi)^2} \ln \left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right] + b_n m_\pi^2$$



Squares full QCD, circles quenched, $\mu = 550 \text{ MeV}$

Instanton Contribution to the Hadronic Masses and Electro-Magnetic Formfactors.

Pietro Faccioli, E.V.Shuryak and A.Schwenk

*Department of Physics and Astronomy State University of New York at Stony Brook/ Stony Brook, New York 11794, USA**

We report the recent results of our study of the instanton contribution to the mass and the electro-magnetic formfactors of light hadrons, in particular of the proton and of the pion.

We begin by presenting a new method that allows to perform parameter-free calculations of selected hadronic observables, in the semi-classical approximation of QCD [1]. Such a method has the advantage to use only results that can be derived from first principles in QCD. It requires neither assumptions about the typical instanton size and density, nor a parameterization of the instanton-instanton interaction. Therefore, all dimensional observables obtained in this way are proportional to direct or inverse powers of Λ_{QCD} .

As a first application, we present a calculation of the pion and of the nucleon dispersion curves. We find excellent agreement with experiment and show that instanton forces generate a light pion and a nucleon with realistic mass ($M_n = 3.9 \pm 0.2 \Lambda_{QCD}^{PV}$).

Next, we analyze the instanton contribution to the pion electro-magnetic formfactor, in the kinematic region which is being investigated at JLAB. This is obtained from a specific combinations of Green functions, in the mixed time-momentum representation.

We show that, in the kinematic region $Q^2 \gtrsim 1 \text{ GeV}^2$, the momentum exchange between quarks, which allows for the recombination of quarks in the final pion state, is mediated predominantly by the field of a single instanton. Our theoretical predictions are completely consistent with the monopole fit, which can be obtained from the vector dominance model. Interestingly enough, such agreement continues up to very high space-like momenta ($Q^2 \sim 10 \text{ GeV}^2$), where the hadronic description breaks down. Our analysis shows that the instanton forces, which are particularly effective in the pseudo-scalar channel, considerably delay the onset of the perturbative regime.

REFERENCES

- [1] P. Faccioli, “*Parameter Free Calculation of Hadronic Masses from Instantons*”, hep-ph/0111322, to appear in Phys. Rev. **D**.
- [2] P. Faccioli and E.V. Shuryak, “*Proton Electro-Magnetic Formfactors in the Instanton Liquid Model*”, hep-ph/0108062, to appear in Phys. Rev. **D**.
- [3] P. Faccioli, A. Schwenk and E.V. Shuryak, “*Instanton Contribution to the Pion Electro-Magnetic Formfactor at $Q^2 > 1 \text{ GeV}^2$* ”, hep-ph/0202027.
- [4] P. Faccioli, A. Schwenk and E.V. Shuryak, “*Instanton Contribution to the Proton Electro-Magnetic Formfactors at $Q^2 > 1 \text{ GeV}^2$* ”, in preparation.

*Electronic address: faccioli@tonic.physics.sunysb.edu

The pion e/m and transition formfactors

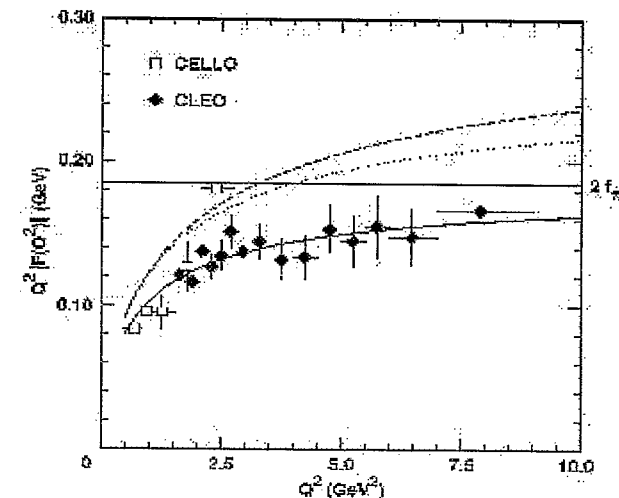
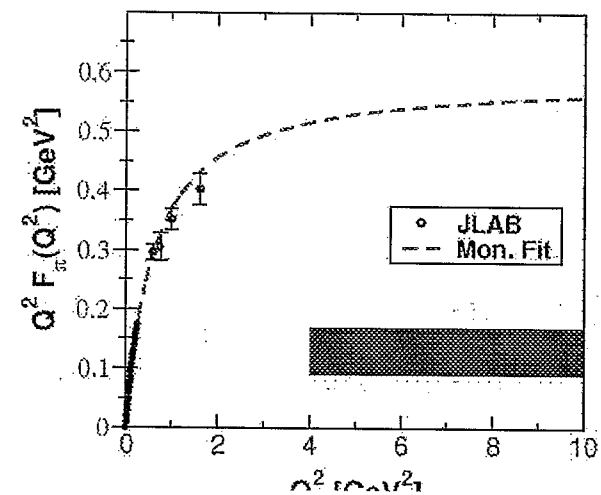
Pion charged e/m formfactor:

Pt-QCD is not reached, in the experimentally accessed region

$\gamma^* \gamma \rightarrow \pi_0$ transition formfactor:

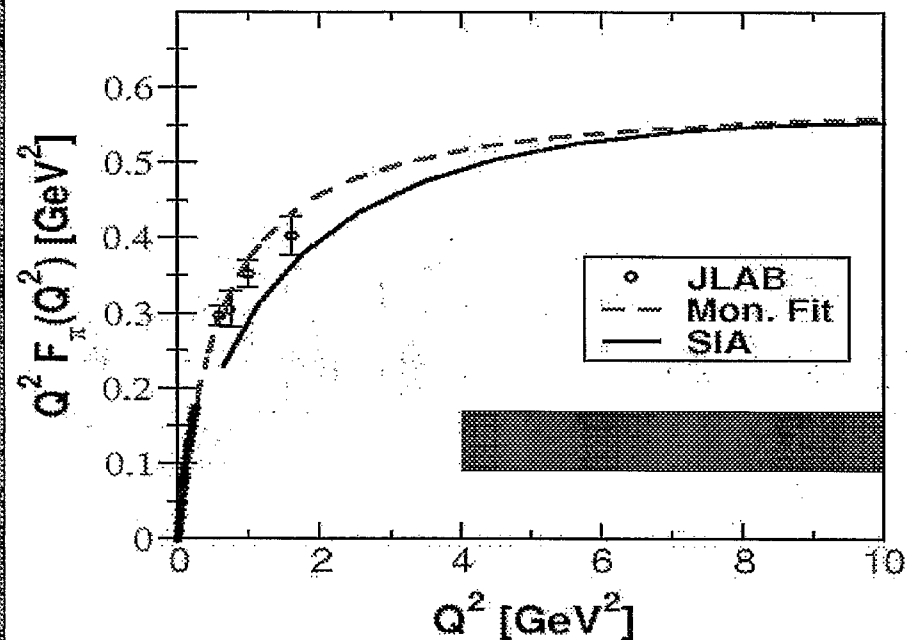
Pt-QCD is reached, at about 2 GeV^2

- * Why such a different behavior?
- * Why, in the E/M FF, vector dominance works up to so large momenta?



Instanton contribution to the Pion E/M formfactor

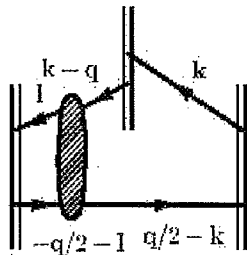
P.Faccioli, A.Schwenk, E.V Shuryak hep-ph/0202027



Instanton contribution completely consistent with the monopole fit (explained by VD) up to $Q^2 = 10 \text{ GeV}^2$.

Instanton contribution is dominant up to very large Q^2 and delays the onset of pQCD.

The leading instanton contribution comes from diagrams like:



NOTICE:

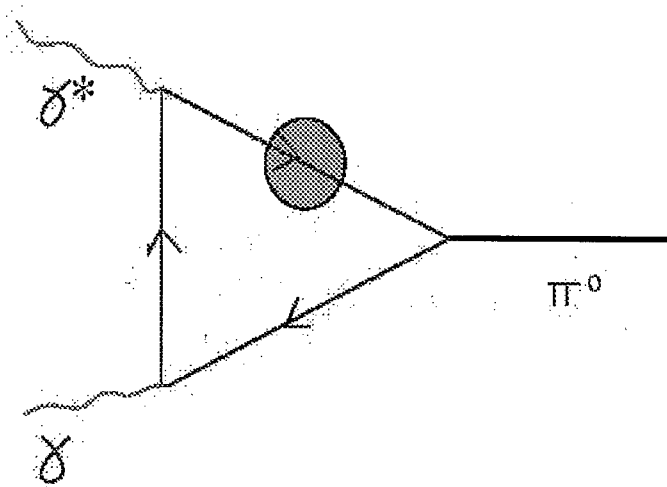
- * one instanton level
- * 2 quarks interact with one instanton!

Instanton contribution to the $\gamma^* \gamma \rightarrow \pi^0$ transition formfactor

The leading instanton contribution to this process is of the type, e.g.:

Notice: only ONE quark interacts with the instanton.

The effect is suppressed by the instanton diluteness w.r.t. the leading contribution to the pion charged FF.



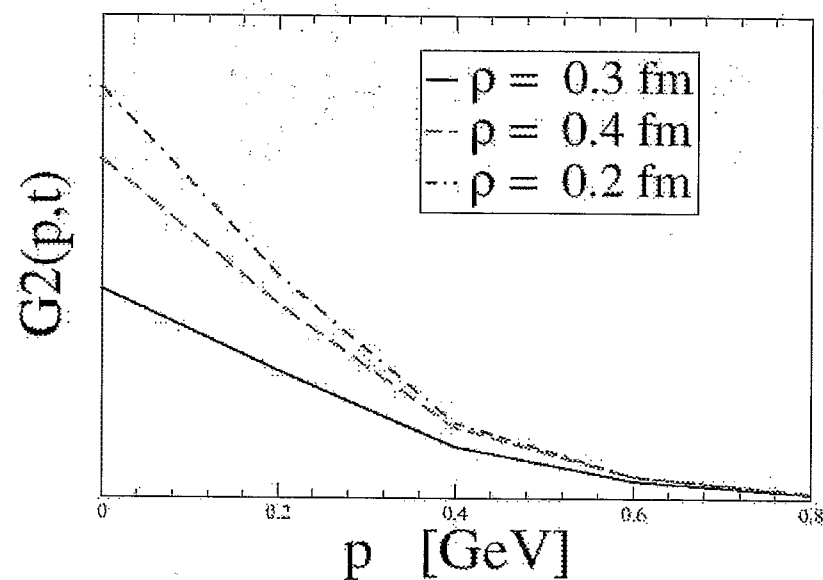
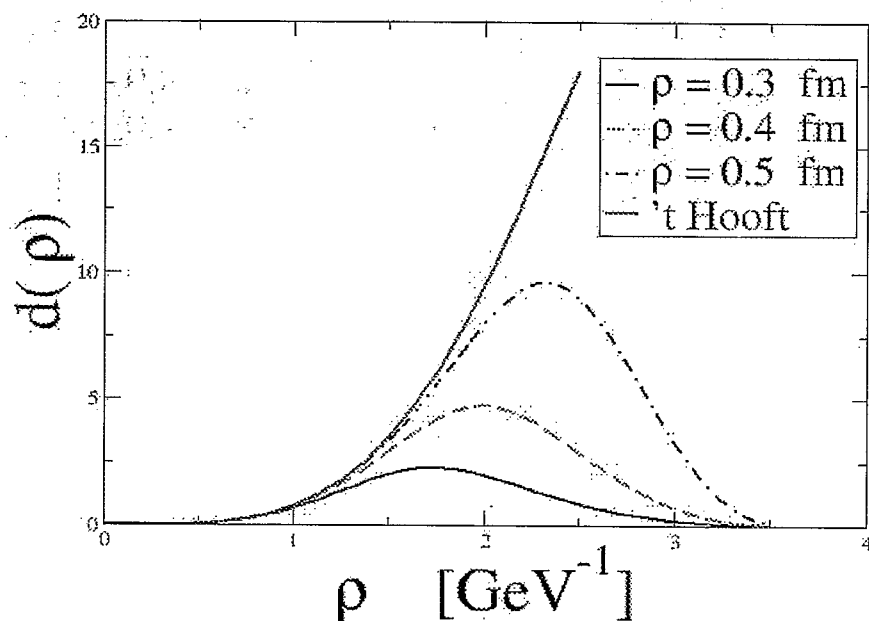
Instanton contribution is about one order of magnitude weaker, in this channel.

This provides an explanation of the early onset of pQCD

Removing the dependence on the instanton size

P.Faccioli, hep-ph/0111322, PRD (2002), in press.

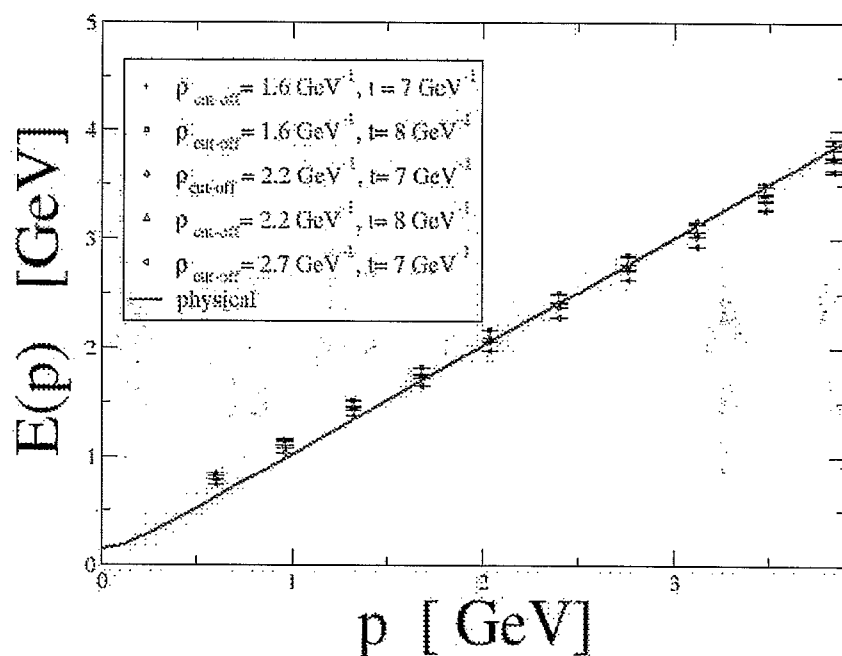
$$G_2^{1271\pi}(t, \mathbf{p}) = \frac{\bar{f}_\pi}{4m^*{}^2} \int_0^\infty d\rho d(\rho) \rho^4 \int_{-\infty}^\infty dz_4 e^{-|\mathbf{p}| \left[\sqrt{z_4^2 + \rho^2} + \sqrt{(t - z_4)^2 + \rho^2} \right]} [\dots smooth \dots]$$



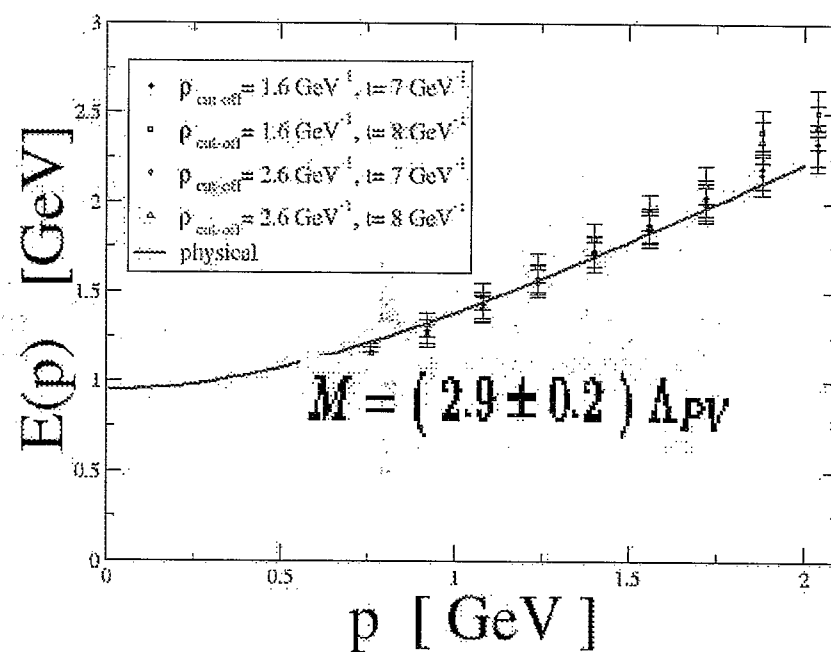
Parameter-free calculation of the pion and nucleon dispersion curves.

P.Faccioli, hep-ph/0111322, PRD (2002), in press.

Pion dispersion curve:



Nucleon dispersion curve:



We proved that the semi-classical, instanton induced interaction generates a light pion and a nucleon pole, with realistic mass.

Dirac Operator Spectrum and Topology with Domain Wall Fermions

Thomas Blum
RBRC

This talk is based on material in two recent papers by the RBC group, hep-lat/0007038 and PPD65, 014504 (2002). It concerns the structure and distribution of the low-lying eigenvalue spectrum of \not{D} for DWF and the consequences for the QCD vacuum (in quenched QCD). Related work is also found in Degrand, Hasenfratz, PRD65014503; Edwards and Heller, PRD014505; Horvath, et al., 014502, Hip, et al. 014506, and Horvath, et al., hep-lat/0201008 (see talk by Keh-Fei Liu on page 75).

III. The eigenvalue spectrum of the Dirac operator. . .

In the continuum chiral symmetry places important constraints on the spectrum:

$$\{\gamma_5, \not{D}\} = 0$$

$$\not{D}^\dagger = -\not{D}$$

So, spectrum of \not{D} comes in pairs,

$$\not{D}\psi_{\pm\lambda} = \pm i\lambda$$

$$\psi_{-\lambda} = \gamma_5\psi_\lambda$$

where λ is real and non-zero

If $\lambda = 0$,

$$\not{D}\psi_0 = 0$$

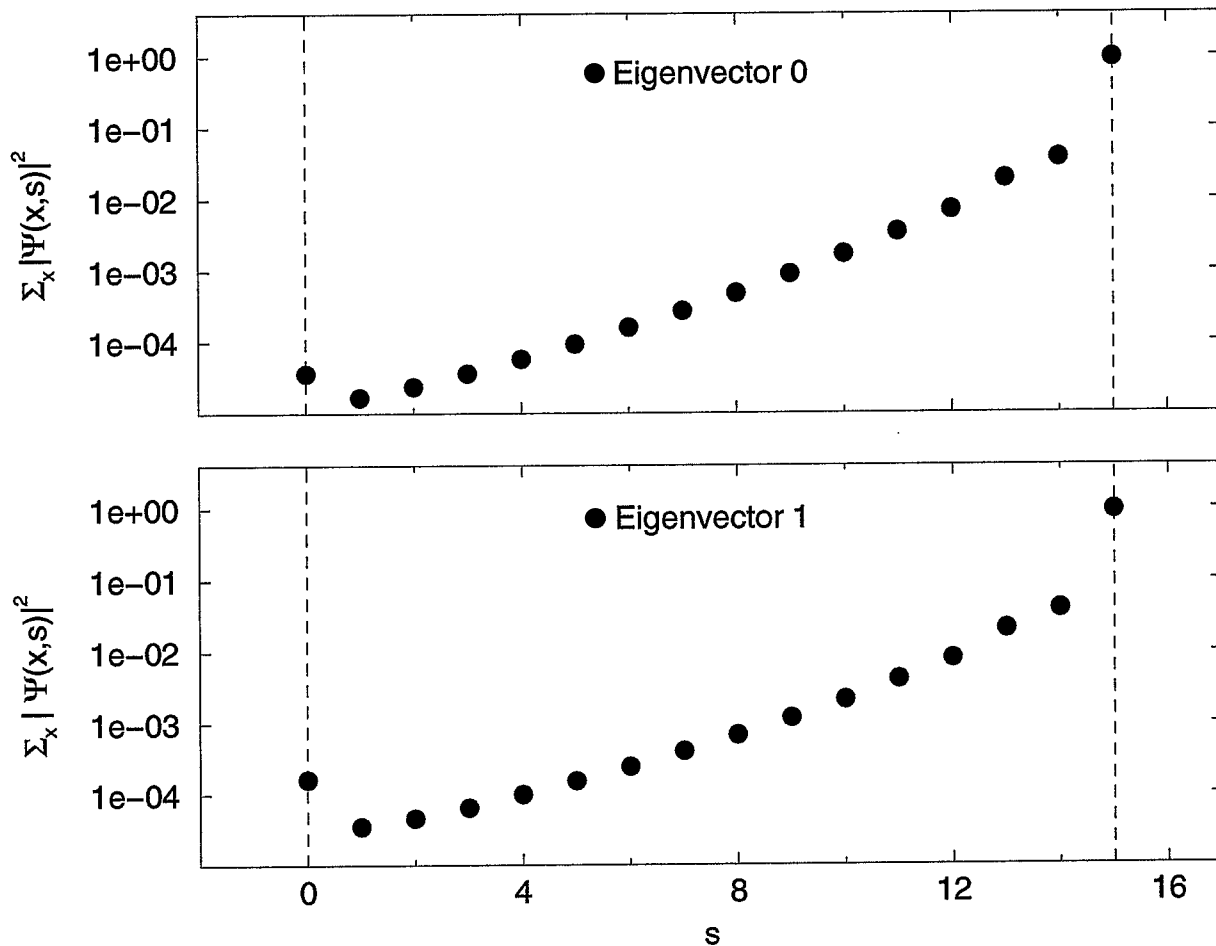
$$\gamma_5\psi_0 = \pm\psi_0$$

and zero-mode are eigenstates of γ_5 (they are chiral).

III. continued

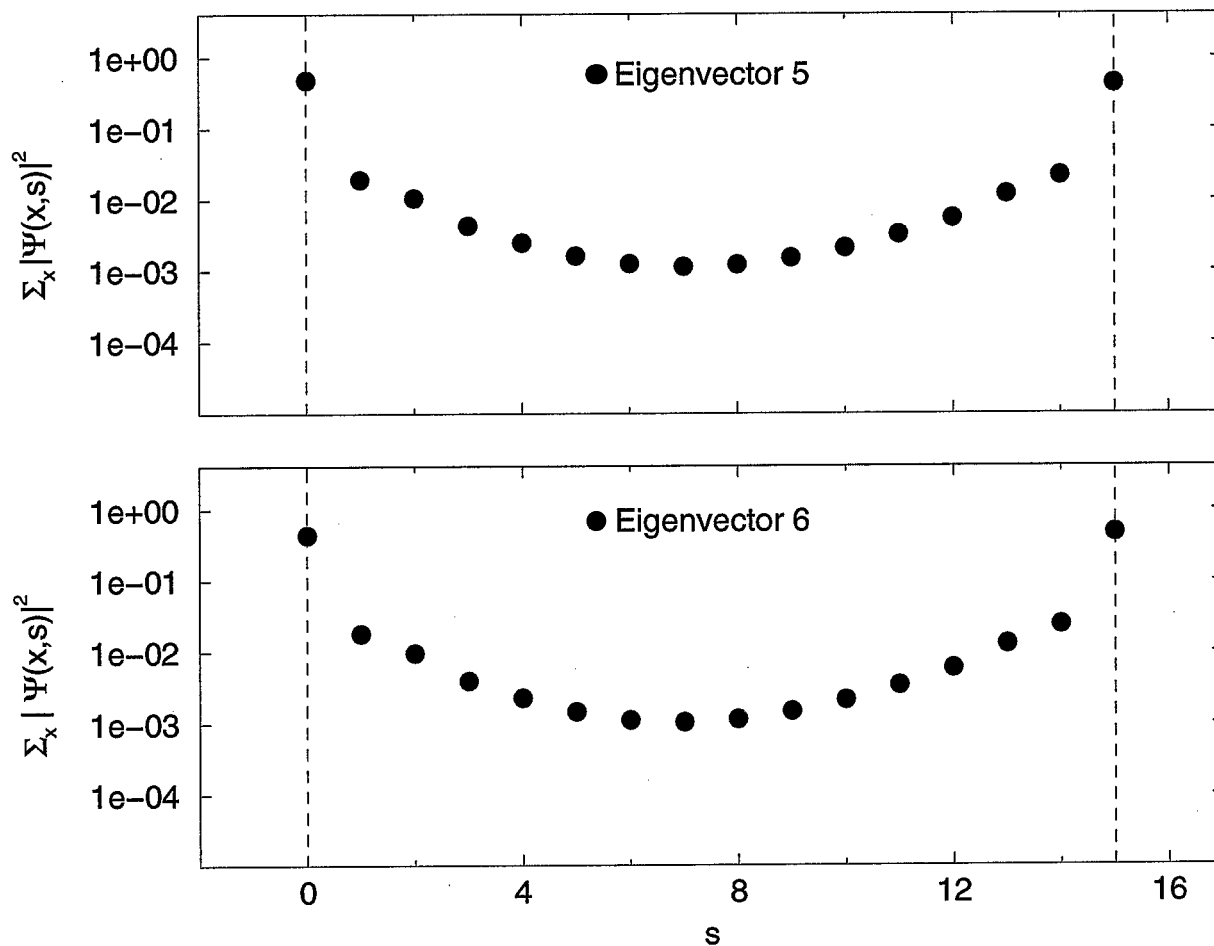
The low energy ($\lambda \ll 1/a, M_5$) eigenvectors and eigenvalues of the 5d DWF Dirac operator represent 4d physical ones.

For example, 2 right-handed chiral zero-modes on a Wilson gauge field:



III. continued

Same gauge field, but 2 non-zero modes:



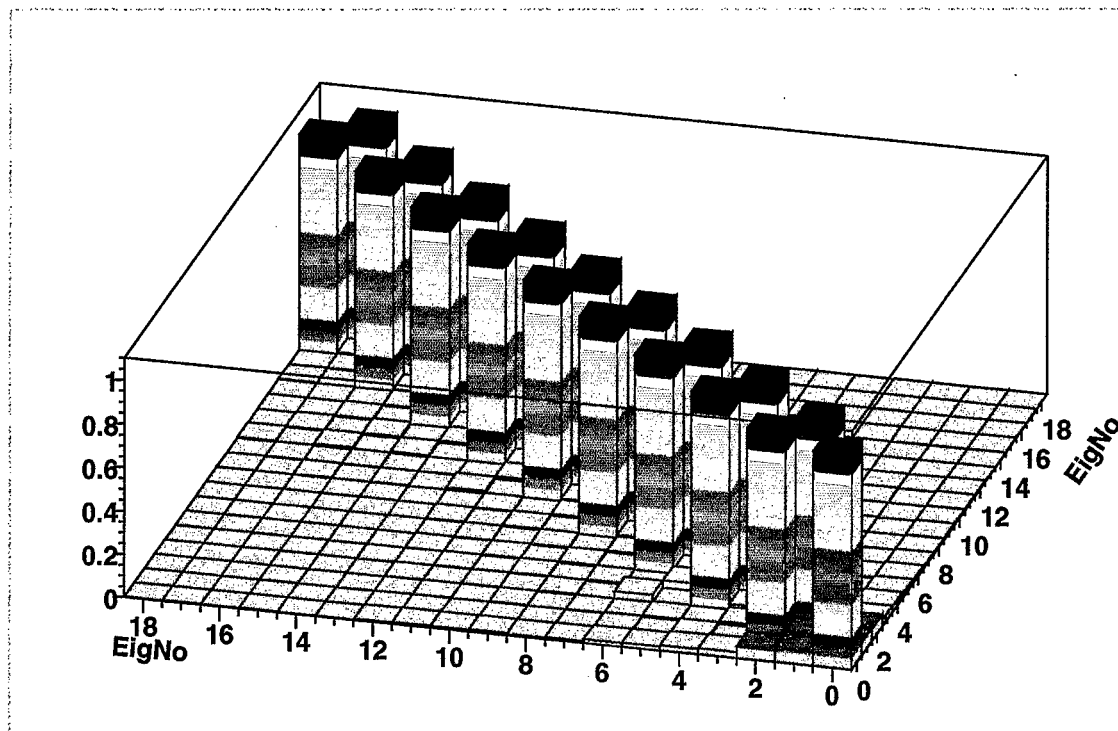
The right- and left- handed components are symmetric about

$$L_s = 7$$

III. continued

Thus we examine the analog of $\langle \lambda_i | \gamma_5 | \lambda_j \rangle$ ($m_q \approx 0$)

$$\langle \Lambda_{H,i} | \Gamma^5 | \Lambda_{H,j} \rangle = \sum_{x \in V} \sum_{s=0}^{L_s-1} \Psi_{\Lambda_{H,i}}^\dagger(x, s) \text{sign}(s - (L_s-1)/2) \Psi_{\Lambda_{H,j}}(x, s)$$



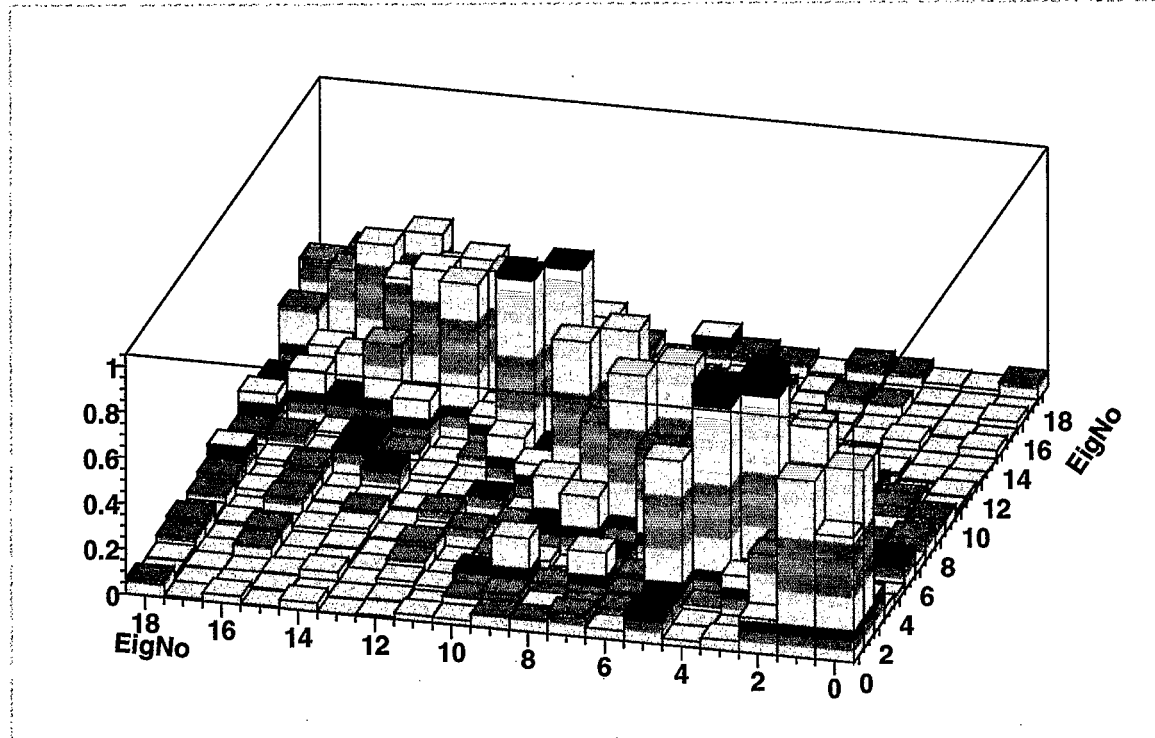
which shows \pm pairs as expected and a single zero mode

III. continued

We also have “complex” configurations

that are **changing topology**

Narayan and Neuberger; Edwards, Heller, and Narayanan



The simple continuum chiral structure is lost.

The measure of such configurations vanishes rapidly

in the continuum limit

Edwards, Heller, and Narayanan

The ensembles contain roughly 10% for Iwasaki and 50% for

Wilson

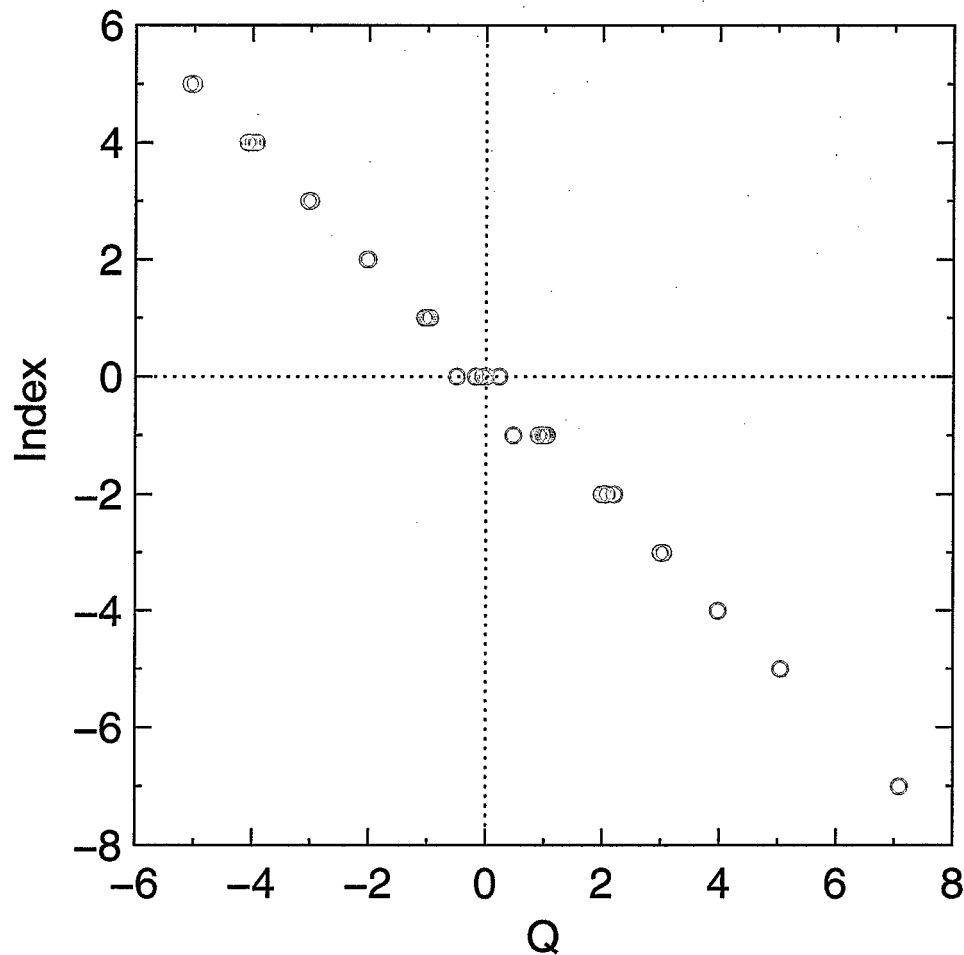
III. continued

The index I of the DWF Dirac operator is obtained by counting zero modes. It agrees quite well with the topological charge

$$Q = \frac{1}{16\pi^2} \int_V F_{\mu\nu} \tilde{F}^{\mu\nu} d^4x$$

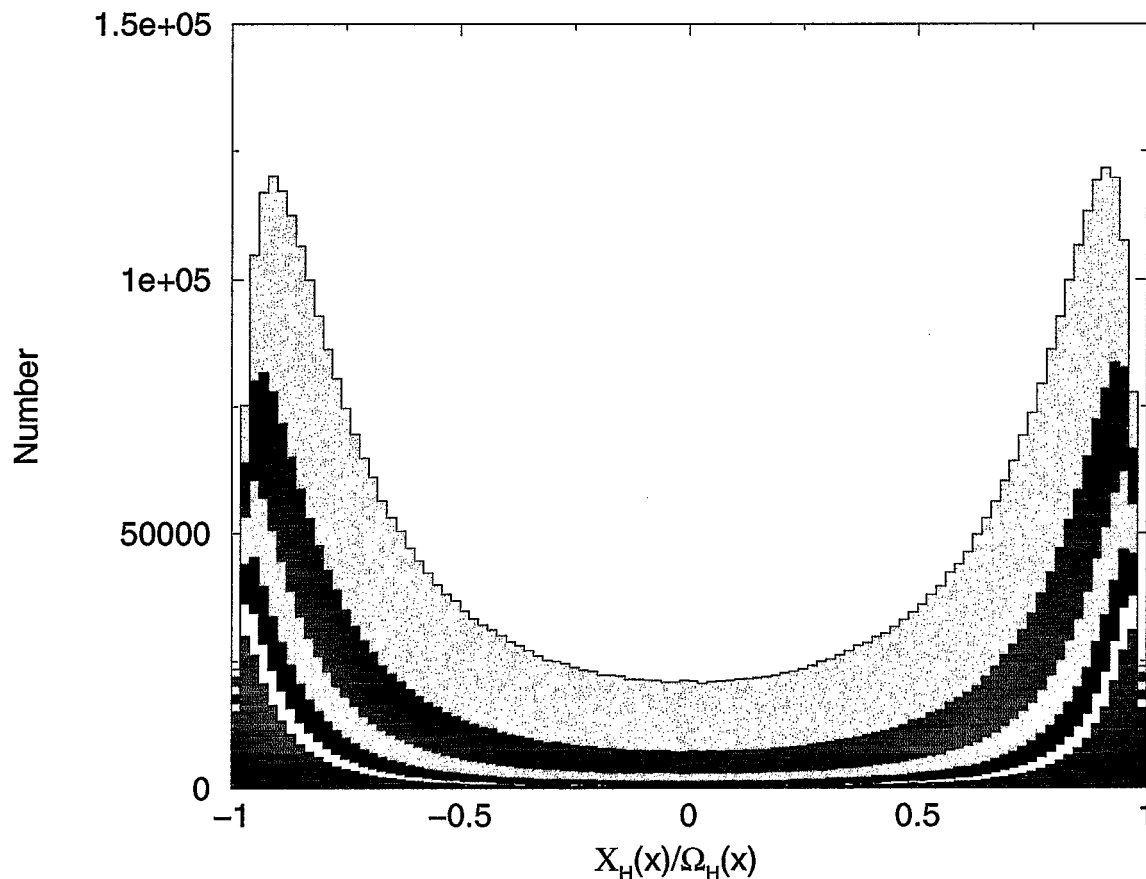
calculated using the method of Degrand, Hasenfratz, and Kovacs

Nucl.Phys. B520 (1998) 301.



III. continued

The local chirality distribution of non-zero modes for the Iwasaki ensemble and domain wall fermions



The double peaked structure indicates that the low-lying modes are *locally chiral* as expected in an instanton model of the QCD vacuum.

Improved domain wall fermions

Konstantinos Orginos

March 29, 2002

We report here our results from our studies of the residual chiral symmetry breaking of domain wall fermions for the 1-loop Symanzik, Iwasaki¹⁾, and DBW2 actions. The DBW2 action was introduced in²⁾ and it was shown by QCD-TARO in³⁾ that it is a good approximation of the RG flow on the two dimensional plane of the plaquette and rectangle couplings.

All the above gauge actions can be written as

$$S_G = \frac{\beta}{3} \left(c_0 \sum_{x;\mu<\nu} P_{\mu\nu} + c_1 \sum_{x;\mu\neq\nu} R_{\mu\nu} + c_2 \sum_{x;\mu<\nu<\sigma} C_{\mu\nu\sigma} \right) \quad (1)$$

where $P_{\mu\nu}$ is the standard plaquette in the μ, ν plane, and $R_{\mu\nu}$ and $C_{\mu\nu\sigma}$ denote the real part of the trace of the ordered product of SU(3) link matrices along 1×2 rectangles and $1 \times 1 \times 1$ paths, respectively. For the 1-loop Symanzik action the coefficients c_0 , c_1 , and c_2 are computed in tadpole improved one loop perturbation theory⁴⁾ While $c_2 = 0$ and $c_0 = 1 - 8c_1$ for both Iwasaki and DBW2, $c_1 = -0.331$ and -1.4069 respectively.

As a measure of the chiral symmetry breaking we use the so called residual mass,

$$m_{\text{res}} = \frac{\langle J_q^5(0) J^5(t) \rangle}{\langle J^5(0) J^5(t) \rangle} \Big|_{t \geq t_{\min}}, \quad (2)$$

where J_q^5 is the mid-point chiral symmetry breaking term which appears in the axial Ward identity of domain wall fermions, and t_{\min} is sufficiently large to avoid short-distance lattice artifacts.

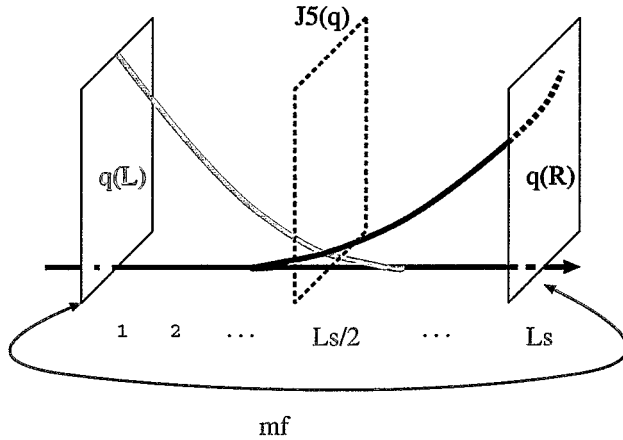
Using the DBW2 action, we have essentially eliminated the problem of residual chiral symmetry breaking for domain wall fermions at 2GeV with $L_s = 16$. Assuming a 440MeV string tension, the bare DBW2 residual mass is about 30KeV. Our results also suggest that the DBW2 action should also be useful for overlap fermions, since it probably makes the approximation to the sign function converge faster by eliminating the small eigenvalues of the Hermitian Wilson Dirac operator.

References

1. Y. Iwasaki, unpublished UTHEP-118 (1983).
2. T. Takaishi, Phys. Rev. **D54**, 1050 (1996).
3. P. de Forcrand *et al.*, Nucl. Phys. **B577**, 263 (2000).
4. M. G. Alford *et al.*, Phys. Lett. **B361**, 87 (1995).

Residual Mass

Measure of Chiral Symmetry Breaking:



$$m_{\text{res}} = \frac{\langle J_{5q}^a(0) J_5^a(t) \rangle}{\langle J_5^a(0) J_5^a(t) \rangle} \Big|_{t \gg 0}$$

$$\langle J_{5q}^a(0) J_5^a(t) \rangle \sim e^{-\lambda_{\tilde{\mathcal{H}}} L_s/2} \quad [\text{Kikukawa \& Noguchi}]$$

Where $\tilde{\mathcal{H}}$ is the 4D Hamiltonian which describes the evolution in the 5th dimension (transfer matrix).

$$\tilde{\mathcal{H}} \sim \gamma_5 D_w^{4d}(M)$$

Where D_w^{4d} is the Wilson fermion action in 4D

In practice:

- $\tilde{\mathcal{H}}$ has many nearly zero eigenvalues.
[Neuberger, Narayanan] [Edwards, Heller, Narayanan] [Hernandez]
- Their frequency decreases rapidly as the continuum limit is approached.
- Improvement: “*Just remove them*”

Improved Chirality \iff Suppressed $\tilde{\mathcal{H}}$ Zero modes

We would like to:

- Identify the gauge configurations that cause \mathcal{H}_t to have Zero modes
- Modify the gauge action so that these configurations get suppressed and hence improve Chiral Symmetry [RBC] [CP-PACS] [Jung et.al.]

Gauge Actions

- Wilson: $S_g = \frac{\beta}{3} \text{Re Tr} \left\langle 1 - \text{square} \right\rangle$

- One loop Symanzik:

$$S_g = \frac{\beta}{3} \text{Re Tr} \left[c_0 \left\langle 1 - \text{square} \right\rangle + 2c_1 \left\langle 1 - \text{rectangle} \right\rangle + \frac{4}{3} c_2 \left\langle 1 - \text{pentagon} \right\rangle \right]$$

c_i computed by 1-loop tadpole improved perturbation theory.

[Lüscher & Weisz Phys.Lett. B158 (1985)]

[Alford et.al. Phys.Lett. B361 (1995)]

- Iwasaki:

$$S_g = \frac{\beta}{3} \text{Re Tr} \left[(1 - 8c_1) \left\langle 1 - \text{square} \right\rangle + 2c_1 \left\langle 1 - \text{rectangle} \right\rangle \right]$$

With $c_1 = -0.331$ computed by perturbative RG blocking.

[Iwasaki (1983)]

- DBW2: Same as Iwasaki but $c_1 = -1.4067$ computed non-perturbatively by RG blocking.

[Takaishi Phys.Rev. D54 (1996)]

The action of a single Instanton on the lattice is:

$$S_g[U^I] = 8\pi^2 \left(1 + C \frac{a^2}{\rho^2} + \mathcal{O} \left(\frac{a^4}{\rho^4} \right) \right)$$

$C > 0 \rightarrow$ Stable instantons (Iwasaki/DBW2)

$C < 0 \rightarrow$ Unstable instantons (Wilson)

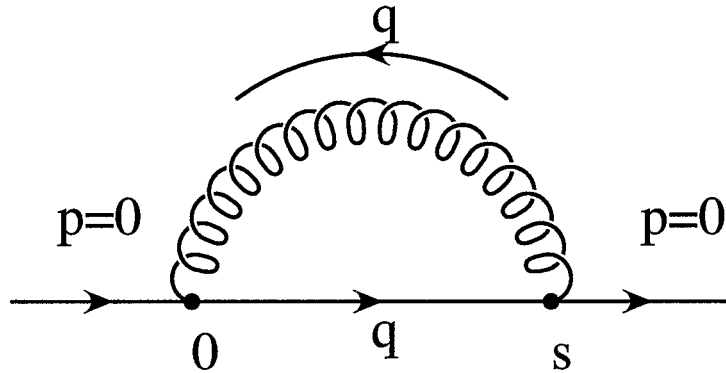
$C = 0$ (Tree level Symanzik)

[Itoh, Iwasaki, Yoshié]

This shows us that the Iwasaki and the DBW2 actions should suppress small instantons and hence improve chiral symmetry.

Another reason inducing chiral symmetry breaking:

[Shamir 2000]



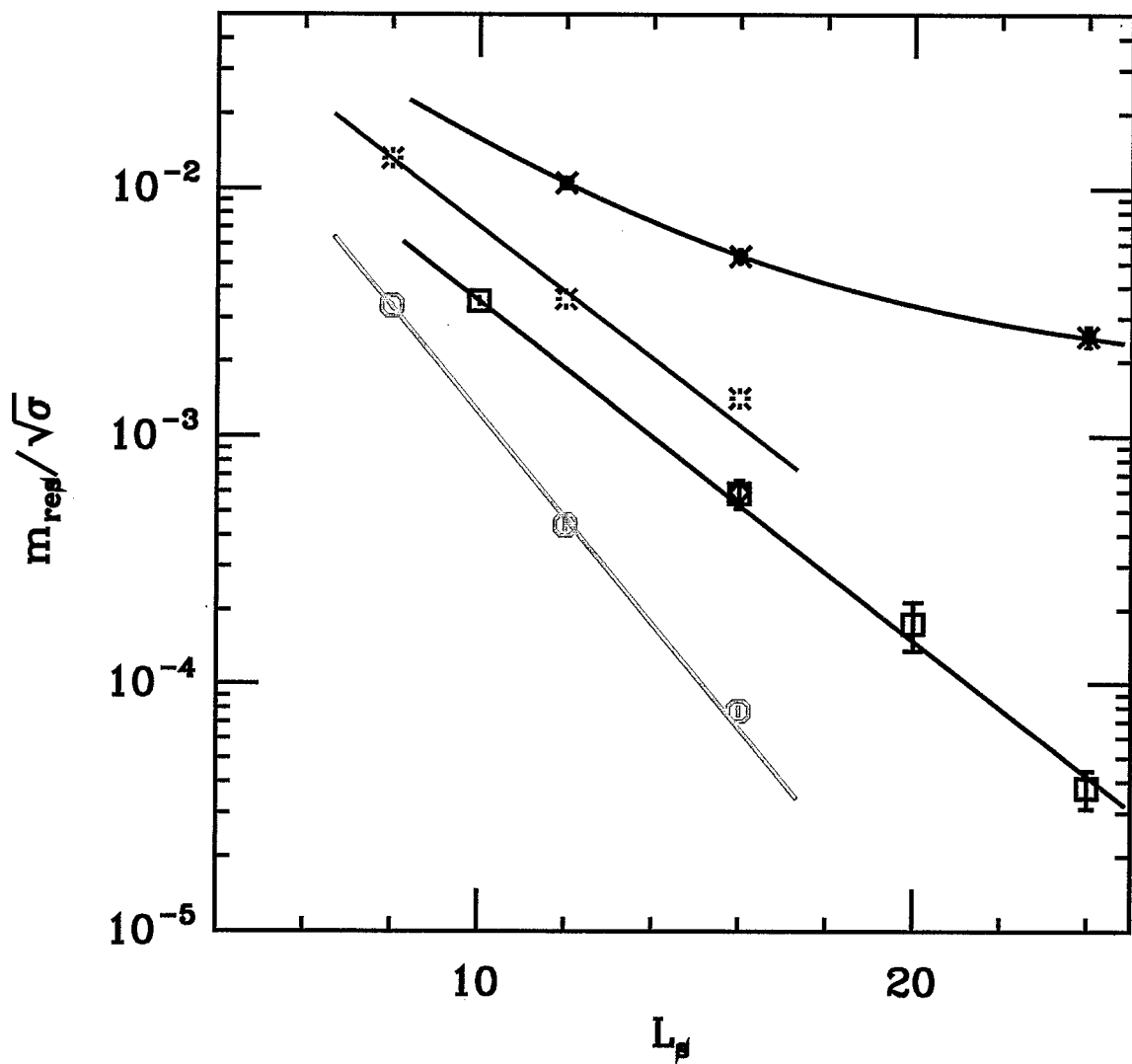
- Low energy quarks \rightarrow Bound to the wall
- High energy quarks \rightarrow Not bound to the wall

Explicit calculation of the above diagram predicts:

$$m_{\text{res}} \sim \frac{1}{L_s^2} \times \left(\frac{1}{2}\right)^{L_s}$$

The Iwasaki and DBW2 action have less high energy fluctuations hence we expect smaller chiral symmetry breaking due to the above mechanism.

m_{res} VS L_s



● DBW2 action ($m_{res}(s) \sim q^s$ with $q \sim .6$).

● Iwasaki action

[CP-PACS Phys.Rev. D63 (2001)]

● Symanzik action.

● Wilson action

[RBC hep-lat/0007038]

Lattice spacing: $a^{-1} \sim 2\text{GeV}$

Lattice QCD with two flavor dynamical domain-wall quarks

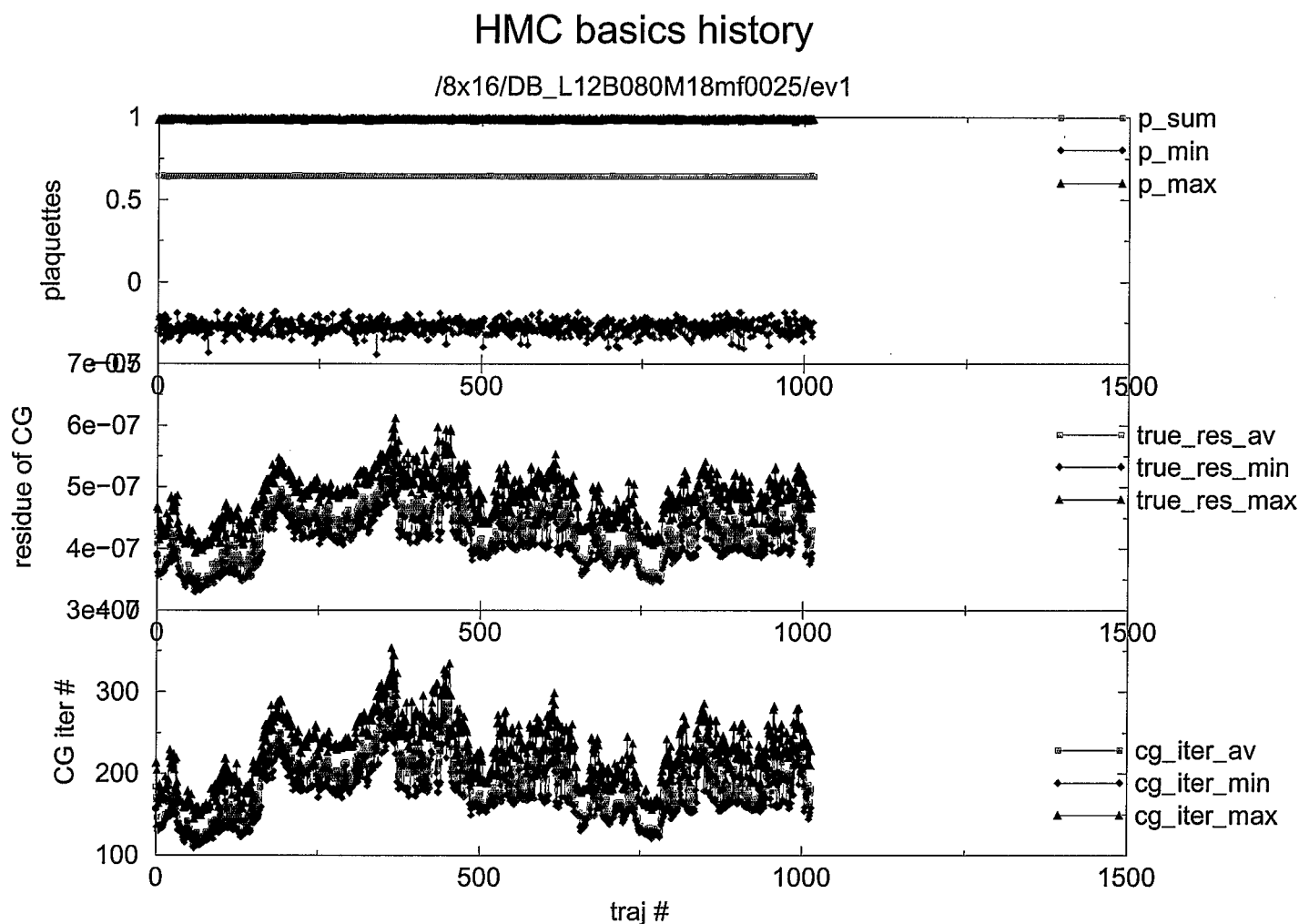
Taku Izubuchi (BNL HET) for Riken-BNL-Columbia
collaboration

- $a^{-1} \sim 2\text{GeV}$
- using improved glue (DBW2,...)
- scale matching with dynamical staggered fermion on deconfined phase
- New force term



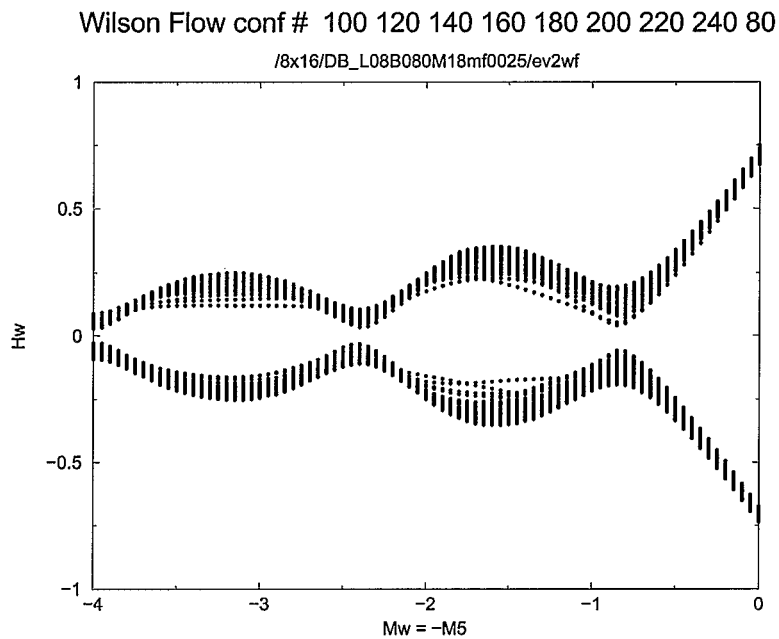
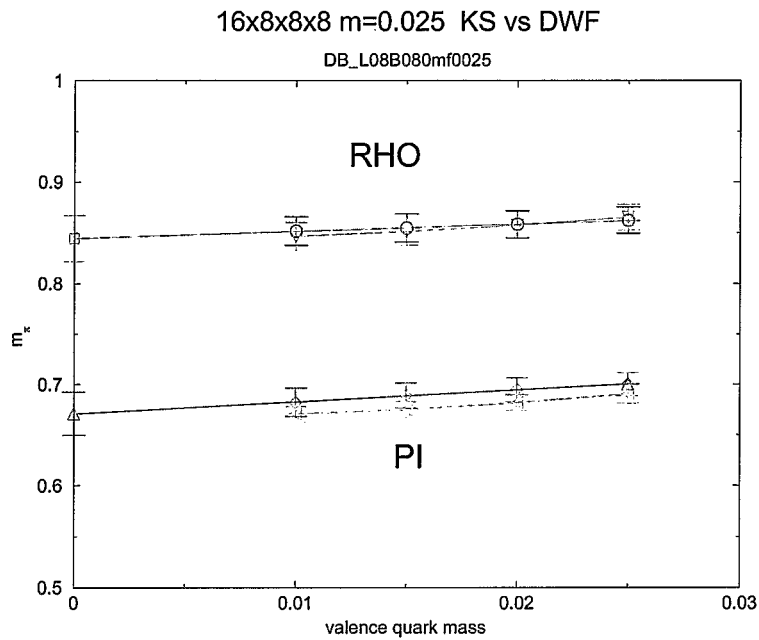
- target scale : $a^{-1} \sim 2 \text{ GeV}$
- Lattice size: $(X,Y,Z,T) = (16,8,8,8), (16,16,16,8), \text{ABC for T-dir}$
- Glueon: WilsonG $\beta = 5.80$, DBW2, $\beta = 0.80$, Iwasaki ...
- quarks: $N_F = 2$, $m_f(dyn) = 0.025, 0.020$
- DWF param.: $M = 1.80, 1.70$, $L_s = 8, 12, (16)$
- HMC param.: $\tau=0.5$, $\Delta\tau = 0.5/\{25, 32, 50\}$

- Preliminary
- num CG. \sim 100--300.
- thermalization after \sim 600--1000 trajs.
- fewer CG for the new force term
- acceptance \sim 70% -- 95%
- elapse time for $8^3 \times 16 \sim$ (an hour / 10 traj for)



Meson masses and Spectrum Flow

$$T = a^{-1}/N_T \sim 250 \text{ MeV}$$

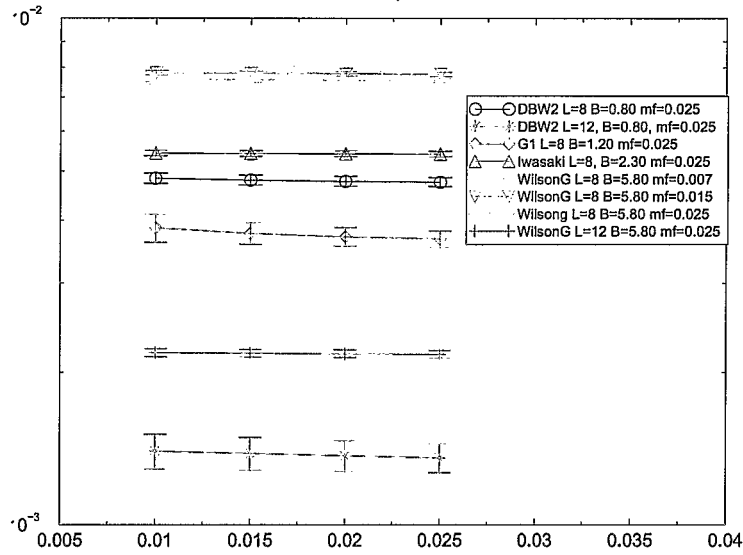


residual masses

mildly depends on Lattice size. a^{-1} varies,

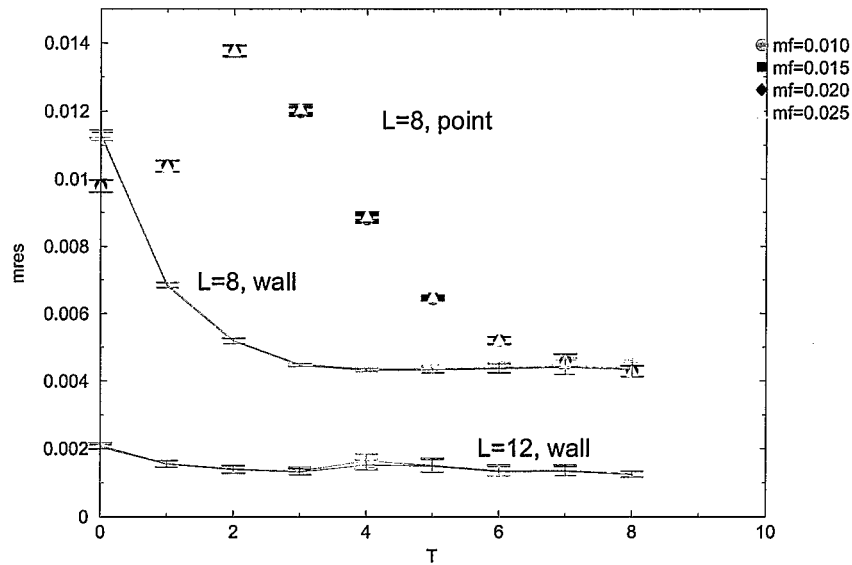
Mres vs mf

8x8x8x16, M=1.80



mres profile

/8x16/DB_L12B080M18mf0025/ev1mL12M18mf0010-0025/hadron/out_100-199



Discussions

- At deconfined phase, on $a^{-1} \sim 2$ GeV lattice
- H_W has a rather clear gap
- 5D eigen vector is sensible (Lego plot)
- m_{res} could be marginally controlled.
- quark mass dependence to chiral condensation is consistently understood. ■
- non zero topological charge in dynamical deconfined phase (?)■
- residual chiral symmetry breaking

$$S_{DWF} = S_{con} + C/P_n(L_s)e^{-\alpha L_s}O(a) + O(a^2)$$

■

- profile of m_{res} shows point source's have severer breaking, ← large momentum modes has less exp suppression

$$\langle J_{5q}(x)P(y) \rangle \sim \exp(-\alpha L_s)$$

α has larger contribution from larger momentum modes

Flavour singlet and exotic mesons from the lattice.

C. Michael

Theoretical Physics Division, Dept. of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK

I show that the difference in mass of flavour singlet and non-singlet mesons can be determined by studying disconnected quark diagrams on the lattice. I present a summary of these contributions for different mesons (hep-lat/0107003) showing that they are only big for scalar and pseudoscalar. I also discuss briefly the technicalities of measuring these disconnected quark diagrams sufficiently accurately.

For a review of pseudoscalar mesons, see hep-lat/0111056.

For scalar mesons, I discuss the quenched case where mixing between quark-antiquark scalar mesons and glueballs can be explored on the lattice. From lattice QCD with 2 flavours of sea quark, only the resulting mixed states are accessible and I present results (hep-lat/0010019 and recent preliminary results). These confirm that significant mixing effects are present. I also discuss the nature of the non-singlet a_0 states.

After a brief review of hybrid mesons, I discuss their decays. It is indeed possible (hep-lat/0201006) to study these decay matrix elements on a lattice. After a brief summary of the restrictions, I present results for hybrid decays for the case of heavy-quark hybrids.

Lattice QCD : Flavour singlet & Exotic States

Chris Michael

University of Liverpool

[UKQCD ; Craig McNeile et al ...]

Lattice QCD's input:

- very m_q , N_f (number of quarks in vacuum loop)
- can't reach m_u, m_d but hope to match to χPT from $m_{\pi}/2$ (say)

Why study flavour singlets?

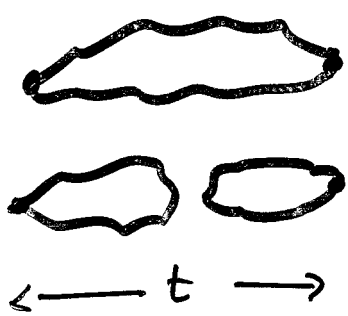
- go beyond naive quark model
- understand OZI rule
- $m_{\eta'}$, O^{++} -glueball sector,
- only indirectly accessible from Quenched ($N_f \rightarrow 0$) QCD.

- OZI for all J^{PC}
- O^{++} mesons
- hybrid meson decays

Flavour singlet mesons

connected $C(t) =$

disconnected $D(t) =$



Quenched

$$C(t) \sim e^{-mt}$$

$$D(t) \sim xt e^{-mt}$$

↑
dipole.

$$\frac{D}{C} \sim xt$$

↑ hope this
in Δm

$N_f = 2$

$$C(t) \sim e^{-mt} \quad \text{non-singlet}$$

$$C(t) + D(t) \sim e^{-m_s t} \quad \text{singlet}$$

$$\frac{D}{C} \sim \frac{d}{c} e^{-\Delta m t} - 1$$

$$(\sim \text{const} - \frac{d}{c} \Delta m t + \dots)$$

$$\Delta m = m_s - m_{NS}$$

so study D/C : OZI violating : related to Δm

OKQCD $m_q \approx m_s$ $N_f = 2$

$$a^{-1} = 1.36 \text{ GeV}$$

(F)

| | | | | |
|---------------|--------------|-------------|-------------|-----------------|
| π^{++} | π^{0+} | π^{++} | π^{+-} | π^{--} |
| ρ^{0-90} | $\eta - \pi$ | $f_1 - a_1$ | $h_1 - b_1$ | $\omega - \phi$ |
| -ve | 0.13(2) | 0.007(13) | 0.001(13) | 0.002(3) |
| | | | | GeV |

caveat :
 • m_q too heavy.
 • decays thus suppressed.

heplat
0107003

hierarchy as experiment.

Only scalar, pseudo scalar have big effect.

Why?

↑
 0^{++} glueball
excitation

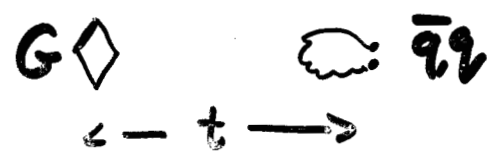
↑
topological charge
density

Scalar Mesons

f_0 where is the glueball?
 what are the $\sigma(700)$ and $f_0(980)$?

a_0 what is the $a_0(980)$?

Quenched:



$\sim 1600 \text{ MeV}$ $\sim 1400 \text{ MeV (SS but !!)}$

(quenched scale uncertain)

Mixing strong at coarse lattice spacing

($E=300 \text{ MeV}$)

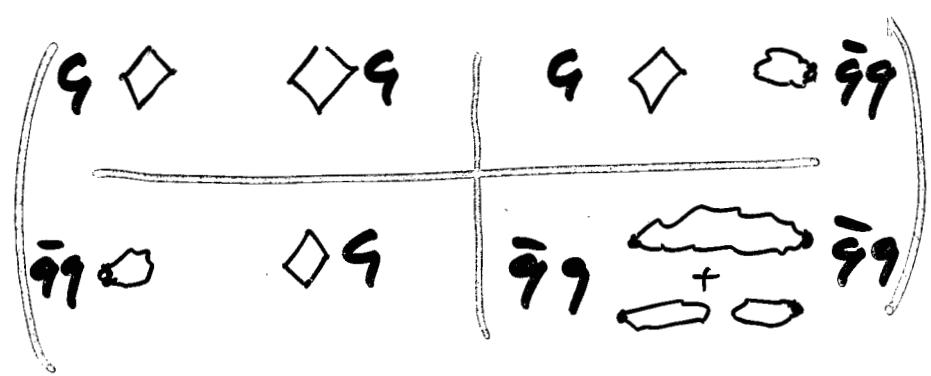
LW claim it gets less as $a \rightarrow 0$

also Weingarten

exploratory studies



$N_f = 2$:



KQCQ

(use 4×4 matrix of correlators) $\rightarrow f_0$ masses (study a_0 also)

SS: $m(a_0) \approx 1.4 \text{ GeV}$
 (so not 980 MeV)

(hep lat 0010019)

(F)
(F)

$N_f = 2$ $s\bar{s}$ quarks:

52

$\alpha' \approx 24 \text{ eV}$

$$\alpha_0 \xrightarrow{1400} \text{---} \xrightarrow{1600} f_0'$$
$$f_0 \xleftarrow{1200} \text{---} \xleftarrow{1000} f_0$$

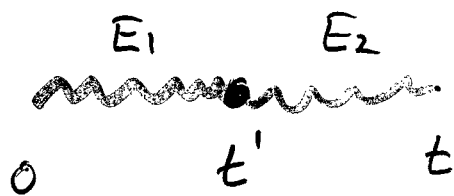
MIXING?

- caveats:
- $O(\alpha^2)$ errors in m_G suppress it
from 1550 \rightarrow 1350 MeV
 - extrapolation to $m_{u,d}$ from m_s may be non-trivial
 - $\pi\gamma$ $\pi\pi$ channels become OPEN

- future:
- include decay channels $\alpha_0 \rightarrow \pi\gamma$
 $f_0 \rightarrow \pi\pi$
 - $\alpha \rightarrow 0$
 - $m_q \rightarrow \text{less}$

Hadronic Decay

Di



$$\approx \int_{t'} e^{-E_1 t'} \propto e^{-E_2 (t-t')}$$

harder to study than weak/e.m. matrix elements
since t' not explicitly known.

so need on-shell study $E_1 \approx E_2$

$$\left. \begin{array}{l} (E_2 - E_1) t \ll 5 \\ \propto t \ll 0 \\ t \gg 0 \end{array} \right\} \begin{array}{l} \text{can study} \\ \text{directly} \\ \text{hep lat} \\ 0201006 \end{array}$$

Note at finite L (space size)
2 body states are discrete ; $k = \frac{2\pi n}{L}$

so study $H \rightarrow \chi_0 \eta$

$G \rightarrow \pi \pi$

$a_0 \rightarrow \pi \eta$

$H \rightarrow \pi \eta$

etc
is feasible

STRING DE - EXCITATION.

Hybrid Meson decay.

CM + CMcNede + P Punsen
hip lab 0201006; PR



lowest spin exotic $J^P = 1^{-+}$
hybrid (HQET: $m_H \rightarrow \infty$)

for $b\bar{b}$ expect $M_H = 10.76(7) \text{ GeV}$
CMetal

Decays:



$B\bar{B}$

$J_2 = 1 \Rightarrow q\bar{q}$ triplet
 $\Rightarrow CP = +$



so forbidden (need $L=1$ in B or \bar{B})

$(Q\bar{Q}) + \text{Meson}(q\bar{q})$

$J_2 = 0$
 $CP = +$

$\hookrightarrow L_2 = 1$

$\therefore \eta\eta'$ D-wave
 $\therefore O^{++}(3)$ P-wave

$H \quad 10.86 \text{ GeV}$
 $\chi_b \quad 9.94 \text{ GeV}$
 $Q = .94 \text{ GeV}$

$H \rightarrow \chi_b O^{++}$ signal $T \approx 80 \text{ MeV}$

$H \rightarrow \chi_b \eta$ no signal $T < 1.5 \text{ MeV}$

Isovector Axial Charge of the Nucleon from Lattice QCD

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c) Institute for Particle and Nuclear Studies, KEK

The iso-vector axial charge g_A of the nucleon is a real good test of lattice QCD towards studying the nucleon structure functions. Indeed, it seems to be an easy quantity for lattice numerical estimation since it requires the lowest moment and zero momentum calculation only for the connected contribution in the SU(2) iso-symmetric case. In contrast to this naive expectation, both quenched and full QCD lattice calculations with Wilson fermion at finite lattice spacing ($a^{-1} \sim 2$ GeV) have not succeeded in reproducing this quantity so well, with underestimation by about 20 % [1]. Recently, the QCDSF Collaboration reported that the systematic error stemming from the finite lattice spacing is not so significant (about 5 %) [2].

Our previous DWF calculation was employed with Wilson gauge action at $\beta = 6.0$ ($a^{-1} \sim 2$ GeV) on a $16^3 \times 32$ lattice, which corresponds to the standard lattice volume ($\sim (1.6\text{fm})^3$) on this subject [1, 2]. We calculate the ratio $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ by virtue of the good chiral properties of DWF ($Z_A = Z_V$) in order to determine the continuum value of g_A in a fully non-perturbative way. We found that $g_A = g_A^{\text{lattice}}/g_V^{\text{lattice}}$ had a fairly strong dependence on the quark mass in the lighter quark mass region [3]. A simple linear extrapolation of g_A to the chiral limit yielded a value that was almost a factor of two smaller than the experimental one.

Here we report our recent study of this issue. In particular, we investigate possible main errors arising from finite lattice volume, especially in the lighter quark mass region. We employ DWF calculation with a RG-improved gauge action (DBW2), which maintains a very good chiral behavior even on coarse lattice ($a^{-1} \sim 1.3$ GeV), in order to perform simulations at large physical volume ($\sim (2.4\text{fm})^3$) [4]. We utilize two lattice sizes ($8^3 \times 24$ and $16^3 \times 32$) to examine the finite volume dependence. The significant large effect on g_A is observed with respect to finite lattice volume [4].

References

- [1] K.F. Liu *et al.*, Phys. Rev. D49, 4755 (1994); M. Fukugita *et al.*, Phys. Rev. Lett. 75, 2092 (1995); M. G"okeler *et al.*, Phys. Rev. D53, 2317 (1996); S. G"usken *et al.*, Phys. Rev. D59, 114502 (1999); D. Dolgov *et al.*, hep-lat/0201021.
- [2] S. Capitani *et al.*, Nucl. Phys. B (Proc. Suppl.) 79, 548 (1999); R. Horsley *et al.*, Nucl. Phys. B (Proc. Suppl.) 94, 307 (2001).
- [3] T. Blum *et al.*, Nucl. Phys. B (Proc. Suppl.) 94, 295 (2001).
- [4] S. Sasaki *et al.*, Nucl. Phys. B (Proc. Suppl.) 106, 304 (2002).

Details of the simulation

Gauge: DBW2 action ($c_1 = -1.4069$)

$\beta = 0.87$, $a^{-1} \approx 1.3$ GeV

two lattice sizes $16^3 \times 32$, $V \approx (2.4 \text{ fm})^3$, 100 configs

$8^3 \times 24$, $V \approx (1.2 \text{ fm})^3$, 400 configs

Fermion: Domain Wall Fermions (quench)

$L_s = 16$, $M_5 = 1.8$

6 quark masses ($m_\pi/m_\rho = 0.62 - 0.89$)

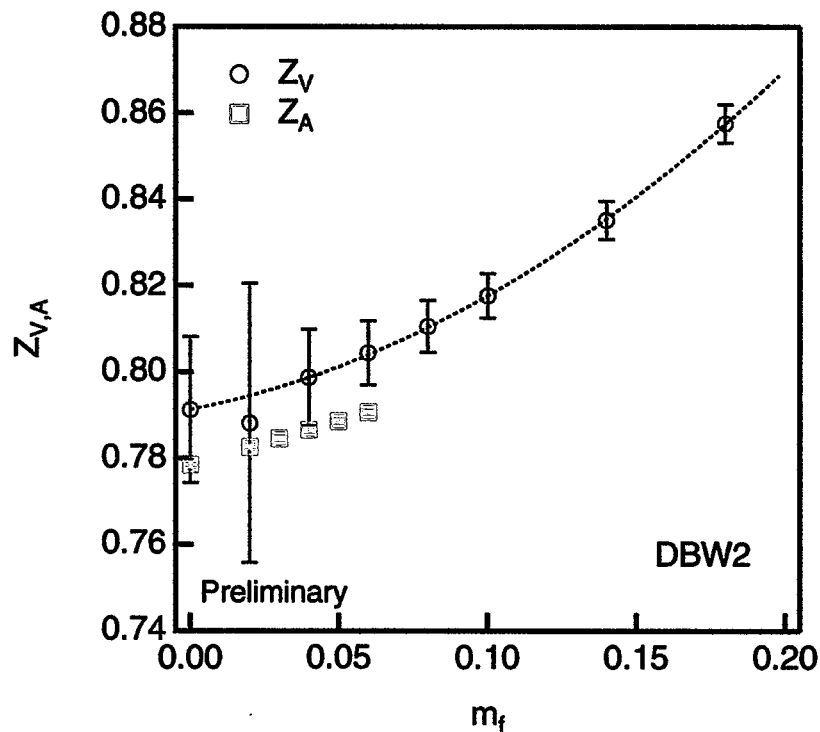
$m_\pi L > 6$ (for the larger volume) and $m_\pi > 500 \text{ MeV}$

Basic results: $m_\rho a = 0.603(20)$, $m_N/m_\rho = 1.30(6)$

$m_{\text{res}} a = 5.7(3) \times 10^{-4}$

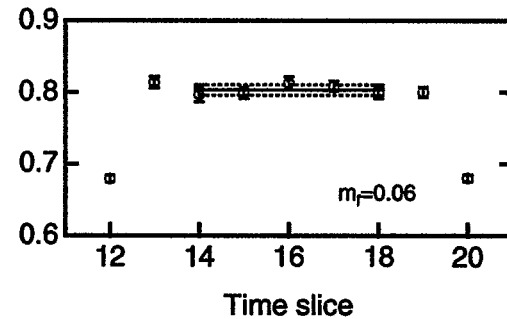
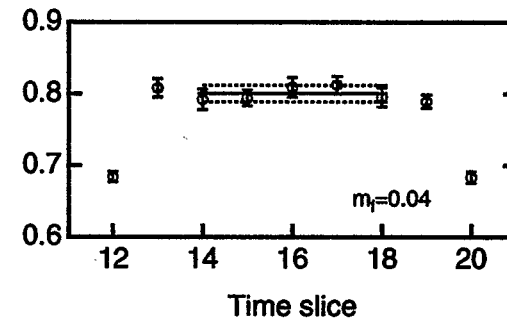
Checking the relation $Z_V = Z_A$

$\beta=0.87$ (DBW2), $16^3 \times 32 \times 16$, $M_5=1.8$



$Z_V = 1/g_V^{\text{lattice}}$ since $g_V = 1$

Z_A from $\langle A_\mu^{\text{conserved}}(t)P(0) \rangle = Z_A \langle A_\mu^{\text{local}}(t)P(0) \rangle$

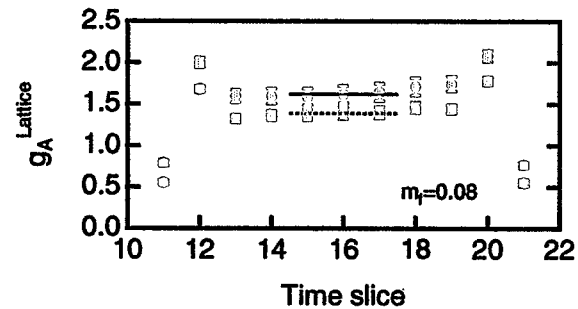
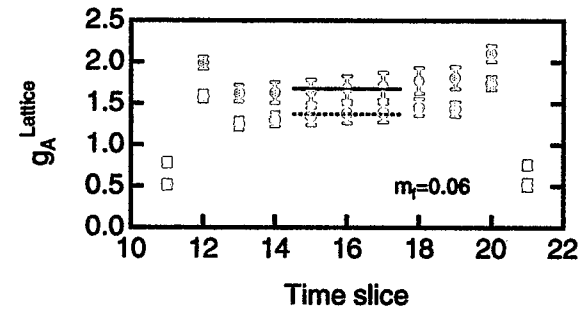
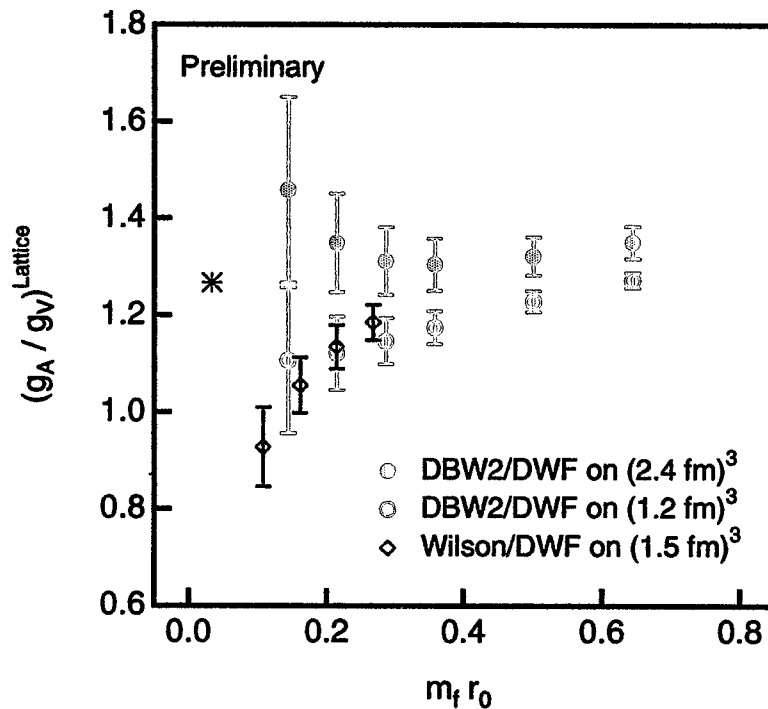


source at $t=11$, sink at 21, current insertions in between

✓ Keep good chiral properties even at coarser lattice.

Large finite volume effect

$\beta=0.87$ (DBW2), $16^3 \times 32 \times 16$ (100 configs) and $8^3 \times 24 \times 16$ (400 configs)

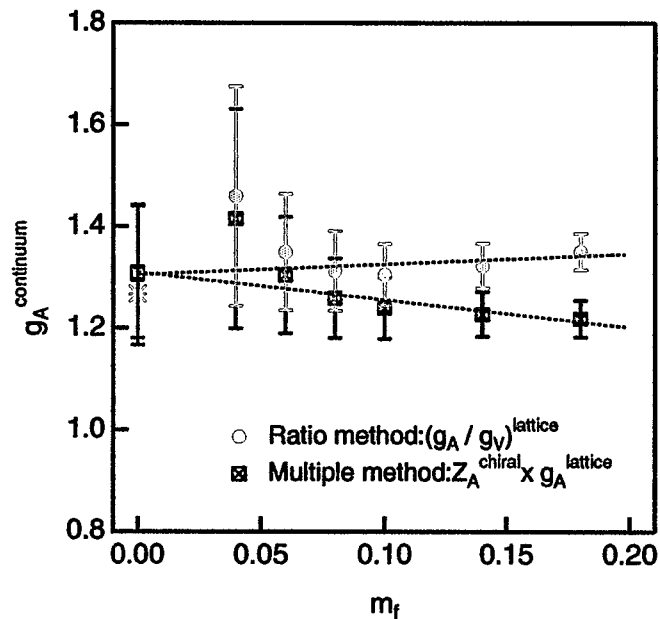


☞ A clear volume dependence is observed

Values of g_A from two methods

Fully non-perturbative determination of g_A

DBW2+DWF, $16^3 \times 32 \times 16$, 100 configs



- Ratio method:

$$g_A^{\text{cont}} = (g_A / g_V)^{\text{lattice}}$$

- Multiple method:

$$g_A^{\text{cont}} = Z_A^{\text{chiral}} \times g_A^{\text{lattice}}$$

- ✓ Mild dependence on m_f

- ✓ Resultant values at $m_f=0$ are consistent; $g_A = 1.30(14)$

Summary

Simulation is employed on coarse lattice ($a \sim 0.15$ fm),
large physical volume $\sim (2.4 \text{ fm})^3$
using DBW2 gauge action and Domain Wall Fermions
 $\beta = 0.87$, two lattice sizes $16^3 \times 32$ and $8^3 \times 24$, $L_s = 16$, $M_5 = 1.8$

- ✓ Relevant three-point functions are well behaved.
- ✓ Confirm that $Z_A = Z_V$ is well satisfied even at coarse lattice.
- ✓ Determine g_A in a fully non-perturbative way.
- ✓ Observe the significant finite volume effect on g_A .

Lightcone nonlocality and lattice QCD

Daniël Boer

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The goal of the spin physics program at RHIC is to measure quantities that describe the spin structure of hadrons in terms of quark and gluon degrees of freedom. Typically these quantities are hadronic matrix elements of operators that are nonlocal along the lightcone; they are functions of lightcone momentum fractions. If one wants to evaluate such quantities using lattice QCD, one needs to resort to evaluating so-called Mellin moments of these functions, which are local operator matrix elements. For example, the first few moments of structure functions have been calculated in this way.

Apart from the cases for which Mellin moments provide a reduction from nonlocal to local operators, there also seem to be a few quantities (all of great interest to high energy spin physics), that do not allow for a straightforward evaluation using lattice QCD. In this talk I focus attention on these quantities and investigate what exactly hampers a lattice evaluation.

The first, hadron spin dependent quantity discussed is a higher twist matrix element called $T(x, S_T)$ [1]. This function possibly forms the origin of the large left-right, single transverse spin asymmetries measured in pion production in hadron-hadron collisions. It appears to be intrinsically nonlocal along the lightcone in any gauge. The question is whether a lattice evaluation of such a matrix element is possible at all. In contrast, other more conventional higher twist matrix elements pose less problems, for example: four quark operator matrix elements or the moments of the twist-3 structure function g_2 (which are quark-gluon correlation matrix elements). The latter function was recently measured to be nonzero by the E155x Collaboration at SLAC and a lattice determination for a particular moment (d_2) has been performed. The advantage of a lattice evaluation of higher twist matrix elements is that one can learn about the magnitude of functions that are not likely to be measured in experiments in the near future.

Although the function $T(x, S_T)$ may not be calculable directly, it may possibly be related to other functions that *are* amenable to lattice evaluation. For instance, the ESGM relation [2] connects the integral (over x) of $T(x, S_T)$ to the quantity d_2 . The derivation of this relation involves an approximation that is unproven, namely assuming an average gluon field strength inside the proton. Whether this yields the dominant contribution remains to be shown. The ESGM relation was derived in the context of the local OPE, but as shown in this talk, it can also be extended to connect the functions of x , instead of the integrals only. This unintegrated ESGM relation may prove useful since the asymmetry A_N involving $T(x, S_T)$ can be measured as a function of x . The issue of whether $T(x, S_T)$ is an oscillating function of x becomes relevant here, since it would allow $A_N(x)$ to be large for certain regions of x , while keeping d_2 small. Especially A_N in the Drell-Yan process (to be measured at RHIC) will be of importance for this issue.

Apart from the lightcone nonlocality problem of $T(x, S_T)$, I also discuss problems related to the polarized gluon distribution $\Delta g(x)$ (its first moment appears in a certain angular momentum decomposition of the proton spin) and to the moments of fragmentation functions. All these quantities frequently appear in factorized and polarized semi-inclusive processes and lattice evaluations/estimations would be more than welcome, but some new ideas will be required.

[1] J. Qiu, G. Sterman, Phys. Rev. Lett. 67 (1991) 2264

[2] B. Ehrnsperger *et al.*, Phys. Lett. B 321 (1994) 121

Qiu-Sterman matrix element $T(x, S_T)$

Some definitions

$$\Gamma_\alpha \equiv \frac{\epsilon_{T\beta\alpha} S_T^\beta \not{n}_-}{2iM \vec{S}_T^2 P^+}$$
$$\langle\langle \dots \rangle\rangle \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | \dots | P, S \rangle$$

such that

$$\begin{aligned} \text{Tr} [\Phi_A^\alpha(y, x) \Gamma_\alpha] &= G_A(x, y) \\ \text{Tr} [\Phi_F^\alpha(y, x) \Gamma_\alpha] &= G_F(x, y) \end{aligned}$$

and define the Qiu-Sterman matrix element $T(x, S_T)$ ($\vec{S}_T^2 = 1$):

$$\begin{aligned} T(x, S_T) &= iM \langle\langle \bar{\psi}(0) \Gamma_\alpha \int d\eta F^{+\alpha}(\eta n_-) \psi(\lambda n_-) \rangle\rangle / P^+ \\ &= 2\pi i M \text{Tr} [\Phi_F^\alpha(x, x) \Gamma_\alpha] / g \end{aligned}$$

One has the problem of dealing with

$$\int d\eta F^{+\alpha}(\eta n_-)$$

Intrinsically nonlocal along the lightcone (even in $A^+ = 0$ gauge)

Not amenable to lattice evaluation: Mellin moments not local

ESGM relation

$T(x, S_T)$ is argued to satisfy the (approximate) ESGM relation

$$\int_{-1}^1 T(x, S_T) dx = -12cM^2 R_0 \int_0^1 x^2 g_2(x)|_{\text{twist-3}} dx$$

Ehrnsperger, Schäfer, Greiner, Mankiewicz, PLB 321 (1994) 121

The derivation was done within the context of the local OPE

ESGM's only approximation is

$$\int d\eta F^{+\alpha}(\eta n_-) \approx F^{+\alpha}(\lambda n_-) \times 2cMR$$

for some λ

If ESGM's approximation is fine, then one can relate the magnitude of SSA's to g_2 ; this is nontrivial and useful, since $\int x^2 g_2(x)|_{\text{twist-3}} dx$ can be (and has been) evaluated using lattice QCD

$$d_2 = 3 \int_0^1 x^2 g_2(x)|_{\text{twist-3}} dx = 2 \int_0^1 x^2 \tilde{g}_T(x) dx$$

E155x data: $d_2 = 0.0032 \pm 0.0016$

This seems to suggest very small SSA (if arising from $T(x, S_T)$)

Unintegrated ESGM relation

Take $P^+ = Mn^+/\sqrt{2}$ in the proton rest frame and take $V = 2cMR$ (c is a constant between 1/3 and 1), then one arrives at

$$gT(x, S_T) = -2cM^2 R x^2 \tilde{g}_T(x)$$

This is an unintegrated ESGM relation, dealing with nonlocal operator matrix elements (holds for each flavor separately)

It can be viewed as approximating

$$\int dy \operatorname{Im} G_A(y, x) \approx xcMR \int dy \operatorname{Re} G_A(y, x)$$

Hence relating the imaginary part of a function to its real part

The question arises whether $\tilde{g}_T(x)$ is an oscillating function, since the unintegrated relation implies similar behavior for $T(x, S_T)$

The x behavior of $T(x, S_T)$

Data (E155x) and the bag model (Jaffe, Ji, PRD 43 (1991) 724) suggest that $g_2(x)|_{\text{twist-3}}$ is an oscillating function of x

Since $g_2(x)|_{\text{twist-3}} = \tilde{g}_T(x) - \int_x^1 \tilde{g}_T(y)/y dy$, $\tilde{g}_T(x)$ itself need not be an oscillating function

In any case $\int T(x, S_T) dx$ can be much smaller than the maximum value of $T(x, S_T)$, but oscillating $T(x, S_T)$ can accomodate small d_2 , but at the same time larger $A_N(x)$ in a specific x range

Lattice evaluation of higher Mellin moments of $x^2 \tilde{g}_T(x)$ would thus be very interesting

$$\int_0^1 x^n g_2(x)|_{\text{twist-3}} dx = \frac{n}{n+1} \int_0^1 x^n \tilde{g}_T(x) dx$$

Conclusions

- Factorized semi-inclusive processes often deal with matrix elements of operators that are nonlocal along the lightcone
- Single transverse spin asymmetries may involve Qiu-Sterman's matrix element $T(x, S_T)$, which cannot be made local by taking Mellin moments
- The ESGM relation offers a solution, but its derivation involves an unproven assumption
- An unintegrated (nonlocal) ESGM relation was obtained
It has the advantage of being a function of x
Lattice evaluation of higher moments of $x^2 \tilde{g}_T(x)$ very useful
- An oscillating $T(x, S_T)$ may produce larger values of $A_N(x)$, while keeping d_2 small
- The angular momentum decomposition in three quark and gluon components is amenable to lattice evaluation
- Fragmentation functions from lattice QCD impossible?

Hard Scattering in the QCD String

Richard C. Brower

- Rutherford's Experiment on Super Strings
- The First String Counter Revolution
- Conformal Red shift in the 5-th Dimension.
- Hard Scattering at Wide Angles
- Soft versus Hard (BFKL) Pomeron

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1. Polchinski and Strassler, hep-th/0109174
"Hard Scattering and Gauge/String Duality",
2. Polchinski and Susskind, hep-th/00112204
"String Theory and the Size of Hadrons"
3. Giddings, hep-th/0203004
"High energy QCD scattering, the shape of gravity on an IR brane and the Friessart Bound"
4. Brower and Tan , hep-th/02xxxxxx
"Hard Scattering in the M-theory dual for the QCD string"

Is QCD a String theory?

QCD has a single expansion parameter:

$$\lambda \equiv 1/N_{color}^2 \simeq 0.1 \quad (1)$$

at fixed mass gap Λ (or equivalently the renormalized 'tHooft coupling $g^2 N$). The only other parameters are m_q/Λ_{qcd} and θ_{qcd} .

The $1/N$ expansion is believed to give

- a leading order spectrum of stable glueballs and stable mesons (including nearly massless pseudo Goldstone pions).
- hard scattering and asymptotic freedom at string tree level.
- Non-perturbative effects $O(\lambda^{-\frac{1}{2}})$: Instantons (Action $\simeq 8N\pi^2/(g^2 N)$), Baryons (Mass $\sim N\Lambda_{qcd}$, ...), etc.

Difficulty with fundamental string for QCD: (i) ZERO MASS STATES: (ii) SUPER SYMMETRY, (iii) EXTRA DIMENSIONS: $4 + 6 = 10$ and (iv) NO HARD PROCESSES!

WIDE ANGLE Scattering ("Rutherford Experiment"): $s, -t, -u \gg 1/\alpha'_s$

$$A_{string}(s, t) \rightarrow \exp \left[-\frac{1}{2}\alpha'_s (s \ln s + t \ln t + u \ln u) \right] \quad (2)$$

but partonic behavior of QCD gives

$$A_{qcd}(s, t) \sim \left(\frac{1}{\sqrt{\alpha'_{qcd} s}} \right)^{n-4} \quad (3)$$

where n =number of "partons" in ext lines (actually lowest twist $\sum_i \tau_i = \sum_i d_i - s_i$) (Conformal up to small asymptotic freedom logs)

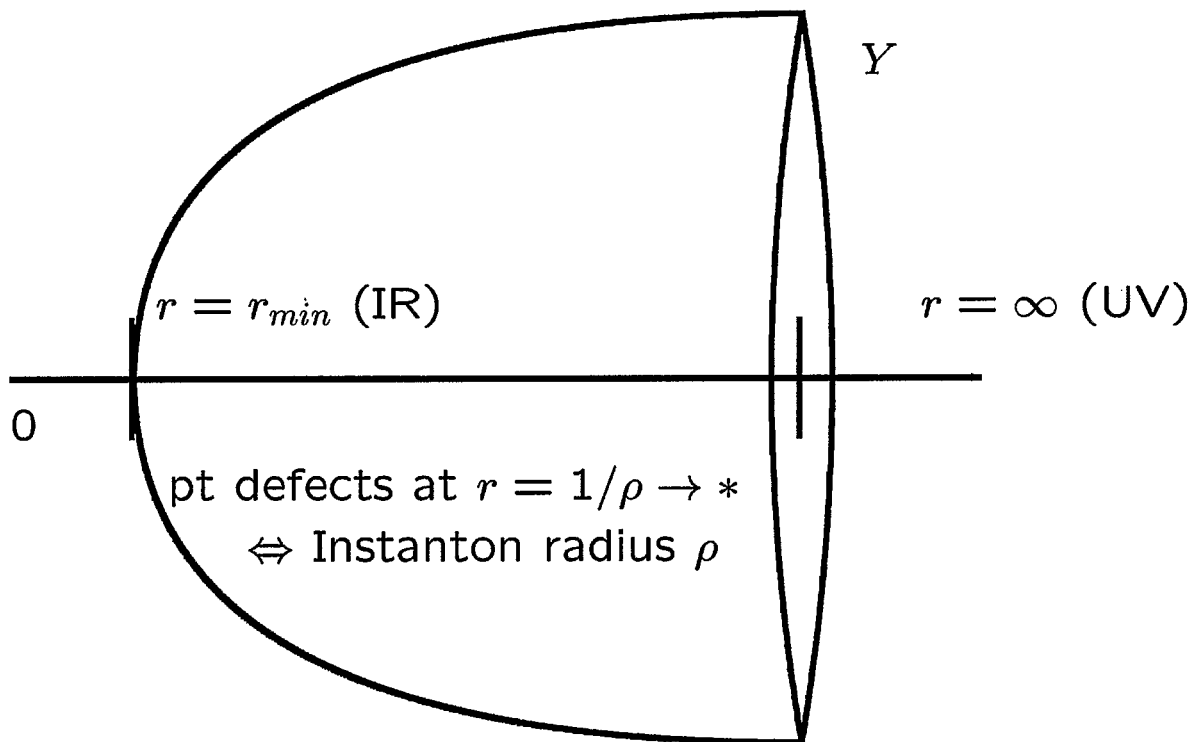
Maldacena's String Counter Revolution

Maldacena ADS/CFT Duality: Weak coupling gravity in $AdS^{d+1} \Leftrightarrow$ Strong coupling (SUSY) CFT in d

Anti-De Sitter metrics, $AdS^{d+1} \times X$, ($\mu = 1, \dots, d$) are solution to low energy closed strings (e.g. gravity). To get the QCD universality class one breaks conformal (and SUSY) symmetry with an IR "cut-off" at $b = r_{min}$

$$ds^2 = \frac{r^2}{R^2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} (1 - b^d/r^d)^{-1} dr^2 + R^2 ds_Y^2$$

where ds_Y^2 is the metric for 5 (or 6) compact variable of 10-d strings theory (or 11-d M-theory).



Wide Angle Scattering

The 2-to- m glueball scattering amplitude $T(p_i)$ in the gravity-dual description is expressed as the 4-d plane wave glueball states $\phi_j(r, Y) \exp[ix_j p_j]$ interacting in via the scattering string(M-theory) amplitude $A(p_i, r_i, Y_i)$ in the 10-d (or 11-d) bulk space (x, t, Y) :

$$T(p_i) = \prod_j \int d\mu_j \phi_j(r_j, Y_j) A(p_1, r_1, Y_1, \dots, p_{m+2}, r_{m+2}, Y_{m+2}) .$$

We now discuss two different approaches to the QCD string that both give the correct parton scaling formula. This is check on the underlining universality of Maldacena's duality:

10-d String theory Approach:

For string theory in a deformed $AdS^5 \times X$ background, Polchinski and Strassler make the following argument. If the bulk scattering is "local" (as it is in strong coupling) then the wide angle strings scatter amplitude $A(p, r_i, Y_i)$ is exponentially suppressed when the local momenta,

$$\hat{p}_s(r) = \frac{R}{r} p , \quad (4)$$

are large relative to the string length

$$\alpha'_s p_s = l_s^2 R p / r_{scatt} > 1 . \quad (5)$$

The scaling is the inverse of the proper distance: $\Delta_s = (R/r) \Delta x$. This is just the standard UV/IR connection in AdS.

Consequently the dominant effect for wide angle is determined by the conformal scaling of the wave function at large r ,

$$\phi_i(r) \sim (r_{scatt}/r_{min})^{-\Delta_4} \simeq \left(\frac{\sqrt{\alpha'_s} p}{\sqrt{r_{min}^2/R^2}} \right)^{-\Delta_4} \sim (\sqrt{\alpha'_{qcd}} p)^{-\Delta_4} \quad (6)$$

in precise agreement with the parton result! The last expression follows from the scaling of the QCD string tension:

$$\alpha'_{qcd} \sim \alpha'_s \frac{R^2}{r_{min}^2}$$

11-d M theory Approach:

Another approach to QCD starts with M-theory in $AdS^7 \times S^4$ with a different conformal scaling $\phi_i(r) \sim (r/r_{min})^{-\Delta_6}$ for glueball wave functions (e.g for scalar fields: $\Delta_6 = 6$ instead of $\Delta_4 = 4$.) However again the correct parton result is found if once one realizes that this is a theory of membranes in which string arise by wrapping the 11-d dimension. The local radius of the 11 dimension $\hat{R}_{11}(r) = rR_{11}/R$ now also determines warped string parameters

$$\hat{l}_s^2 = \hat{R}_{11}(r) l_p^3 \quad \text{and} \quad \hat{g}_s^2(r) \hat{R}_{11}(r) / l_p^3. \quad (7)$$

These additional warping factors precisely reproduce the parton results!

$$\left(\frac{r_{scatt}}{r_{min}} \right)^{-\Delta_6^{(i)}} \sim \left(\frac{\sqrt{\alpha'_s} p}{\sqrt{r_{min}^3/R^3}} \right)^{-\frac{2}{3}\Delta_6^{(i)}} \sim (\sqrt{\alpha'_{qcd}} p)^{-\frac{2}{3}\Delta_6^{(i)}} \quad (8)$$

Note that the last expression in M-theory requires the scaling relation:

$$\alpha'_{qcd} \sim \alpha'_s \frac{R^3}{r_{min}^3}$$

The 3rd power is a consequence of minimal (3-d) membrane world volumes in 11-d versus (2-d) strings world surface in 10-d.

Soft vs Hard Regge Scattering

Similar arguments can be applied to the Regge limit: $s \gg -t$.

$$T(s, t) = \int_{r_{min}}^{\infty} dr \mathcal{K}(r) \beta((R^3/r^3)t) (\alpha'_s)^{\alpha_s(0) + \alpha'_s(R^3/r^3)t}. \quad (9)$$

The dominant scattering takes place at large r , giving rise to a BFKL-like Pomeron with almost flat “trajectory” (actually a cut in the j -plane)

$$T(s, t) \sim (\alpha'_s)^{\alpha_s(0)} / (\log s)^{\gamma+1} \quad (10)$$

The IR region in the bulk $r \simeq r_{min}$ gives a normal soft Regge pole with slope $\alpha'_{qcd} \sim \alpha'_s R^3 / r_{min}^3$

$$T(s, t) \sim \exp[+\alpha'_s t \log(s)] (\alpha'_s)^{\alpha_s(0)} \quad (11)$$

Actually we can not really see the Regge pole without a better computational control over the IR region. But the basic form is consistent with the expectation that soft behavior approaches the exponential damping of naive strings, consistent with a “form factor” with a divergent radius in impact parameter:

$$\langle X_{\perp}^2 \rangle \simeq \alpha'_{qcd} \log(s) \sim \alpha'_s \log(\text{No. of d.o.f}) \quad (12)$$

Relativistic Heavy Quarks on the Lattice

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Abstract

Lattice QCD should allow quantitative predictions for the heavy quark physics from first principles. Up to now, however, most approaches have based on the nonrelativistic effective theory, with which the continuum limit can not be taken in principle. In this paper we investigate feasibility of relativistic approaches to the heavy quark physics in lattice QCD. We first examine validity of the idea that the use of the anisotropic lattice could be advantageous to control the $m_Q a$ corrections. Our perturbative calculation, however, reveals that this is not true. We instead propose a new relativistic approach to handle heavy quarks on the isotropic lattice. We explain how power corrections of $m_Q a$ can be avoided and remaining uncertainties are reduced to be of order $(a\Lambda_{\text{QCD}})^2$.

$O(a)$ improvement for massive quark

1. Off-self improved action

$$S_q^{\text{imp}} = \sum_x \left[c_3 \mathcal{O}_3 + c_4 \mathcal{O}_5 + \sum_{A=a}^b c_{5A} \mathcal{O}_{5A} + \sum_{A=a}^f c_{6A} \mathcal{O}_{6A} + \dots \right]$$

$$\text{dim.3 : } \mathcal{O}_3(x) = \bar{q}(x)q(x),$$

$$\text{dim.4 : } \mathcal{O}_4(x) = \bar{q}(x) \not{D} q(x),$$

$$\text{dim.5 : } \mathcal{O}_{5a}(x) = \bar{q}(x) D_\mu^2 q(x),$$

$$\mathcal{O}_{5b}(x) = i \bar{q}(x) \sigma_{\mu\nu} F_{\mu\nu} q(x),$$

$$\text{dim.6 : } \mathcal{O}_{6a}(x) = \bar{q}(x) \gamma_\mu D_\mu^3 q(x),$$

$$\mathcal{O}_{6b}(x) = \bar{q}(x) D_\mu^2 \not{D} q(x),$$

$$\mathcal{O}_{6c}(x) = \bar{q}(x) \not{D} D_\mu^2 q(x),$$

$$\mathcal{O}_{6d}(x) = i \bar{q}(x) \gamma_\mu [D_\nu, F_{\mu\nu}] q(x),$$

$$\mathcal{O}_{6e}(x) = \bar{q}(x) \not{D}^3 q(x),$$

$$\mathcal{O}_{6f}(x) = \bar{q}(x) \Gamma q(x) \bar{q}(x) \Gamma q(x),$$

...

$$\text{dim.}(3l+n) : \mathcal{O}_{3l+n}(x) = \bar{q}(x)^l \Gamma^l D_\mu^n q(x)^l$$

c_{iA} 's can be mass-dependent \rightarrow higher dimensional op.

With on-shell condition($\not{D} + m = 0$)

$$\bar{q}(x)^l \Gamma^l D_\mu^n q(x)^l \sim (ma)^n (a\Lambda)^{(3l-4)}$$

\rightarrow only $l = 1$ (fermion bilinear) is needed for $O(a)$ improvement.

Parameters(no tuning)

$$Z_m \leftrightarrow m_0 \Leftarrow \text{hadron mass}$$

$$Z_q \rightarrow \nu_t = 1 \Leftarrow \text{pole mass tuning}$$

$$r_t = 1(\text{ irrelevant, Wilson parameter})$$

Relevant parameters(tuning is necessary)

$$\nu_s = 1 + \sum_{n=2}^{\infty} (ma)^n (\nu_s)_n (g^2) \equiv 1 + f_{\nu_s}(ma)$$

$$r_s = r_t + \sum_{n=1}^{\infty} (ma)^n (r_s)_n (g^2) \equiv 1 + f_{r_s}(ma)$$

$$c_{E,B} = c_{\text{SW}} + \sum_{n=1}^{\infty} (ma)^n (c_{E,B})_n (g^2) \equiv 1 + f_{c_{E,B}}(ma)$$

Note that the expansions in ma converge if ma is small enough.

~~$ma \rightarrow \infty$ behaviours?~~

FNAL quark action

- ν , c_E , c_B are relevant parameters in the action
- r_s is removed from the action by the change of the variable:

$$q(x) \rightarrow e^{da\gamma_i D_i} q(x).$$

However d term is necessary for the matrix elements. $\rightarrow r_s \Leftrightarrow d$

- Kinetic mass is used for the hadron masses
- If ν_s is not tuned(as the clover action), $(ma)^2$ error appears.
Pole mass is better (?).

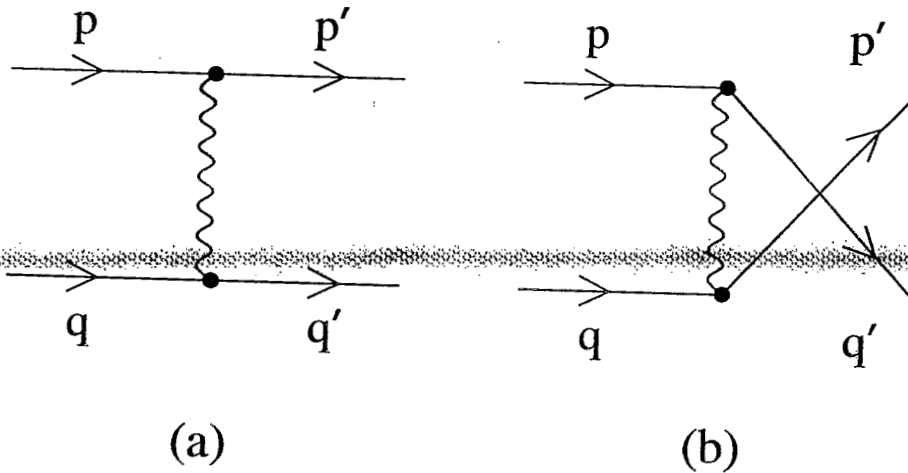
Determination of parameters at tree level

Renormalization and Improvement condition

Fermion propagator

$$S_F^{\text{lat.}}(p_0, \vec{p}) = \frac{1}{Z_q} \frac{-i\gamma_0 p_0 - i \sum_i \gamma_i p_i + m_p}{p_0^2 + \sum_i p_i^2 + m_p^2} + (\text{no pole terms}) + O((p_i a)^2)$$

Quark-quark scattering amplitude



$$T = \text{continuum amplitude} + O((p_i a)^2)$$

Renormalization

$$m_p = \log \left| \frac{m_0 + r_t + \sqrt{m_0^2 + 2r_t m_0 + 1}}{1 + r_t} \right|,$$

$$Z_m = \frac{m_p}{m_0},$$

$$Z_q = \cosh(m_p) + r_t \sinh(m_p).$$

Improvement coefficients

$$\nu_s = \frac{\sinh(m_p)}{m_p},$$

$$r_s = \frac{\cosh(m_p) + r_t \sinh(m_p)}{m_p} - \frac{\sinh(m_p)}{m_p^2},$$

$$c_E = r_t \nu_s,$$

$$c_B = r_s.$$

- Propagator & scattering amplitude give the same ν_s and r_s .
→ r_s is relevant.
- 1-loop calculation is in progress.
- Non-perturbative determination
 $\nu_s \leftarrow$ kinetic mass = pole mass
 $r_s \leftarrow$ kinetic mass of heavy-heavy & heavy-light ?
 $c_{E,B}$? (No axial Ward-Takahashi identity)

Scaling violation for heavy quarks

Scaling violation

$$\begin{aligned}
 a\Lambda_{QCD} \sum_{n=0}^{\infty} C_n^W(g^2, \log a)(m_Q a)^n & \quad \text{Wilson with pole mass tuned} \\
 a\Lambda_{QCD} \sum_{n=1}^{\infty} C_n^C(g^2, \log a)(m_Q a)^n & \quad \text{massless clover with pole mass tuned} \\
 (a\Lambda_{QCD})^2 \sum_{n=0}^{\infty} C_n^F(g^2, \log a)(m_Q a)^n & \quad \text{fully tuned}
 \end{aligned}$$

Again expansions converge for small ma .

Large mass behaviour

Heavy-light decay constant (example)

static limit

$$\frac{f_{HL}}{\Lambda_{QCD}} = \sqrt{\frac{\Lambda_{QCD}}{m_{HL}}} [w_0 + (a\Lambda_{QCD})^2 w_2 + O((a\Lambda_{QCD})^3)]$$

fully tuned action

$$\frac{f_{HL}}{\Lambda_{QCD}} = v_0(\Lambda_{QCD}/m_Q) [1 + \underbrace{(a\Lambda_{QCD})^2 v_2(m_Q a)}_{\text{relative scaling violation}} + O((a\Lambda_{QCD})^3)]$$

\Downarrow

$$\begin{aligned}
 v_2(m_Q a) & \rightarrow O(1) \quad \text{as } m_Q a \rightarrow 0 \\
 & \rightarrow w_k \sim O(1) \quad \text{as } m_Q a \rightarrow \infty
 \end{aligned}$$

$v_2(m_Q a)$ is finite in the $m_Q a \rightarrow \infty$ limit

and even a mild function of $m_Q a$!

Non-perturbative Renormalisation with DWF

Chris Dawson [RIKEN-BNL Research Center]

March 21, 2002

Abstract

In this talk I gave a brief overview of the results of applying the technique of non-perturbative renormalisation as developed by the Rome-Southampton group to the renormalisation of local bilinear and four-quark operators using the Domain Wall Fermion action.

The good chiral symmetry of Domain Wall Fermions means that the quark propagator in a fixed gauge is off-shell improved, with discretisation errors starting at $O(a^2)$. This was demonstrated by showing results for the trace of the inverse quark propagator and contrasted to the result for on-shell improved Wilson fermions which suffer from appreciable $O(a)$ errors.

The results for local bilinear operators were then shown. The calculated renormalisation factors obey the relations that chiral symmetry predicts, and agree well with one-loop, tadpole improved perturbative calculations. However, for the standard choice of external quark momenta the extraction of the pseudo-scalar renormalisation constant has a large systematic effect due to contamination with pion poles.

Results from the more complicated case of the renormalisation of the $\Delta S = 2$ Hamiltonian were then shown. These showed no evidence that different operator mixing between different chiral multiplets, a result which allows practical calculation of the Hadronic parameter B_K with Domain Wall Fermions. The contamination with pion poles, however, was shown to be a significant effect, but one that could be completely circumvented by a different choice of external momenta.

Non-perturbative Renormalisation

- Apply renormalisation conditions directly on lattice Green's functions with off-shell quark states [Martinelli *et al*, NPB445]
 - in a fixed gauge
 - at large momenta
- In principle an all-orders calculation of the renormalisation coefficients.
- Also allows lattice perturbation theory to be avoided
 - slow convergence (can mean-field improve).
- Requires a “window” in momenta
 - not too high - lattice artifacts
 - not too low - low energy effects
- In reality systematic effects from either side of the window may be taken into account
 - Improvement
 - Operator product expansion

Trace of S^{-1} : Lattice form

- The actual form of the $Tr(S^{-1})$ in a lattice calculation will be contaminated by

1. Low energy, spontaneous chiral symmetry effects [Politzer, NPB117] which at lowest order in $1/p^2$ look like

$$C_1 \frac{\langle \bar{q}q \rangle}{p^2}$$

2. High energy, discretisation effects. The leading effects are from defining the quark field [Becirevic et al, hep-lat/9909082]

$$q_{\text{ren}} = Z_q^{1/2} (1 + b_q m_f) \{ a c_q (\not{D} + m_{\text{ren}}) + c_{\text{NGI}} \not{D} \} q_{\text{bare}}$$

these terms break chiral symmetry

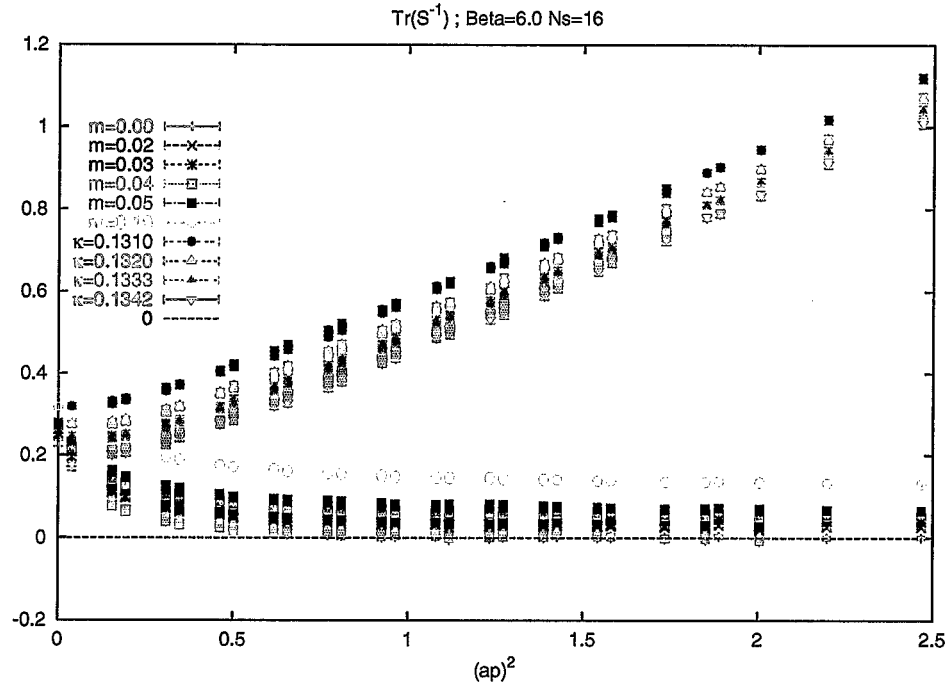
– suppressed by $O(am_{\text{res}})$.

- Expected lattice form:

$$\frac{1}{12} \text{Tr}(S_{\text{latt}}^{-1}(ap)) =$$

$$\begin{array}{ll} \text{Low Energy Corrections} & \dots + \frac{a^3 \langle \bar{q}q \rangle}{(ap)^2} C_1 Z_q \\ \text{The Renormalised Mass} & + Z_m Z_q \{ am_f + am_{\text{res}} \} \\ \text{Lattice Artifacts} & + 2 (c_{\text{NGI}} Z_q - c'_q) (ap)^2 + \dots, \end{array}$$

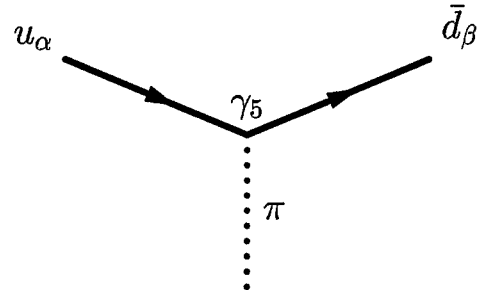
Trace of S^{-1} : Data



- Compare Wilson (APE) and DWF (RBC) results.
- No sign of explicit chiral symmetry breaking effects.
- Spontaneous chiral symmetry breaking effects small at scale of 2GeV .

Pion Poles

- The gauge configurations are non-perturbative and allow for the propagation of light mesons. ie. effective interaction

$$C \{ \bar{\pi} \bar{d} \gamma_5 u + \bar{u} \gamma_5 d \pi \}$$


The diagram illustrates a vertex interaction. Two solid lines with arrows pointing towards a central vertex are labeled u_α and \bar{d}_β . From this vertex, a vertical dotted line extends downwards, labeled with π . The vertex itself is marked with γ_5 .

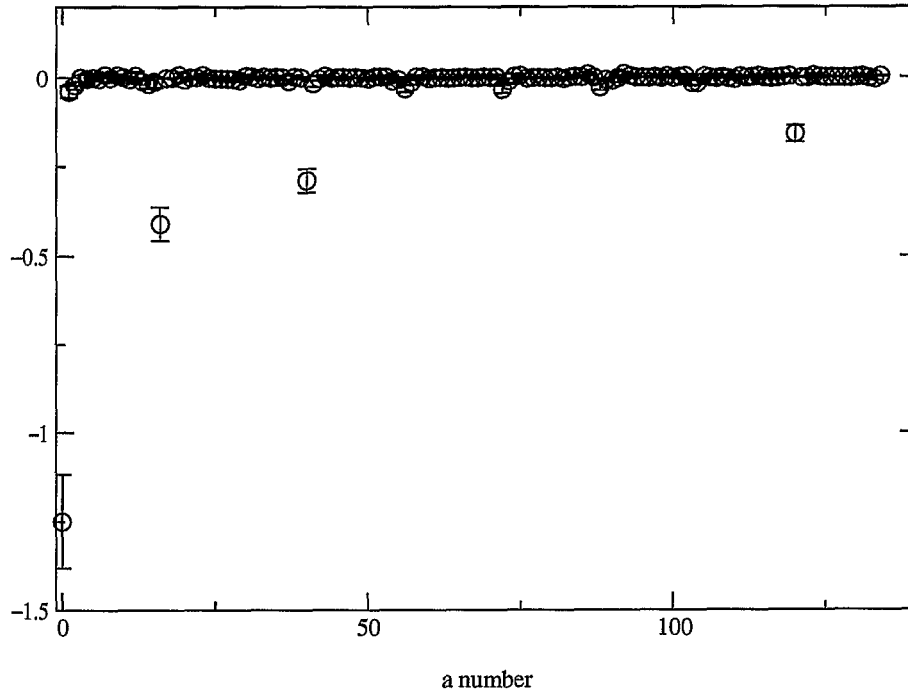
- Pion Propagator:

$$\frac{1}{(p-q)^2 + M_\pi^2}$$

- $(p-q) = 0$ and $M_\pi^2 \propto m$. Have a $1/m$ pole for every pion propagator.
- Solutions:
 - Fit to the poles.
 - Use a different momenta configuration (later).

Distinct momenta

- $(MF^{-1})_{34}$



- New momenta configuration for external states

$$s(p)\bar{d}(q)s(p)\bar{d}(q)$$

- Various momenta configurations for $m_f = 0.04$.
- The four points with appreciable magnitude have $p = q$.

One loop average momentum of singlet parton densities in the Schrödinger Functional scheme

Filippo Palombi

in collaboration with

R. Petronzio and A. Shindler

- Singlet operators on the lattice
- Schrödinger Functional
- Gauge invariant sources
- One loop perturbative expansion
- Renormalization in the SF scheme
- Analysis and results

Ref.: hep-lat/0203002

**One loop average momentum
of singlet parton densities
in the Schrödinger Functional scheme**

A non perturbative computation of the evolution of singlet parton densities without gauge-fixing requires gauge invariant sources in order to probe singlet operators. Within the Schrödinger Functional scheme, a quark source can be defined in terms of functional derivatives with respect to the quark boundary fields and a gluon source — which is the novel part of our work — can be defined in terms of path ordered products of gauge links, connected to the time boundaries. We adopt this definition and perform a one loop lattice computation of the renormalization constants of the twist-2 operators that correspond to the second moment of singlet parton densities. We use the Wilson action, the Feynman gauge and perform our calculation at zero renormalized quark mass.

We check our calculation verifying that the non physical divergences, present at single diagram level, cancel out when summing over all diagrams, and that the physical divergences lead to the known one loop anomalous dimensions.

This calculation fixes the connection between the lattice SF scheme, where a non perturbative evaluation of the absolute normalization of singlet parton densities can be made at low energy, and the \overline{MS} scheme where one can extract the experimental values, determining the matching coefficients which are needed to jump from one renormalization scheme to the other.

This computation puts the premises for a lattice non perturbative calculation of the amount of gluon in a hadron from first principles. The new definition of a gauge invariant gluon source might also be used for a novel definition of α_s , or for further studies of non perturbative aspects of the gluon propagation.

Singlet operators

- Moments of singlet structure functions are related, through the operator product expansion, to hadronic matrix elements of two kind of twist-2, gauge invariant, local operators of the form

$$O_{\mu_1 \dots \mu_N}^q = \frac{1}{2N} \bar{\psi} \gamma_{[\mu_1} \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_N]} \psi$$

$$O_{\mu_1 \dots \mu_N}^g = \sum_{\rho} \text{tr} \{ F_{[\mu \rho} \overleftrightarrow{D}_{\mu_2} \dots \overleftrightarrow{D}_{\mu_{N-1}} F_{\rho \mu_N]} \}$$

- On the lattice, irreducible representations may require combinations of operators with different Lorentz indices
[M.Göckeler et al., Phys.Rev. D54 (1996)]. For the second moment, a possible choice is ($N = 2$)

$$O_{12}^q(x) = \frac{1}{4} \bar{\psi}(x) \gamma_{[1} \overleftrightarrow{D}_{2]} \psi(x) \quad (1)$$

$$O_{12}^g = \sum_{\rho} \text{tr} \{ F_{[1\rho}(x) F_{\rho 2]}(x) \} \quad (2)$$

Gauge invariant sources

Quark gauge invariant source [Karl Jansen et al., Phys.Lett. B372 (1996)]

- Let's introduce functional derivatives with respect to the boundary fields:

$$\zeta(\mathbf{x}) = \frac{\delta}{\delta \bar{\rho}(\mathbf{x})}, \quad \bar{\zeta}(\mathbf{x}) = \frac{\delta}{\delta \rho(\mathbf{x})}$$

$$\zeta'(\mathbf{x}) = \frac{\delta}{\delta \bar{\rho}'(\mathbf{x})}, \quad \bar{\zeta}'(\mathbf{x}) = \frac{\delta}{\delta \rho'(\mathbf{x})}$$

They can be considered as quarks on the boundary and can be used to define the operators

$$S_q(\mathbf{p}) = a^6 \sum_{\mathbf{y}, \mathbf{z}} e^{i\mathbf{p} \cdot (\mathbf{y} - \mathbf{z})} \bar{\zeta}(\mathbf{y}) \Gamma \zeta(\mathbf{z})$$

$$S'_q(\mathbf{p}) = a^6 \sum_{\mathbf{y}', \mathbf{z}'} e^{i\mathbf{p} \cdot (\mathbf{y}' - \mathbf{z}')} \bar{\zeta}'(\mathbf{y}') \Gamma \zeta'(\mathbf{z}')$$

- It can be shown that these operators are invariant under a gauge transformation of the SF. Choosing $\Gamma = \gamma_2$ and $p_1 \neq 0$, such a source can be used to probe the operators (1,2)

Gauge invariant sources (2)

Gluon gauge invariant source

- A gauge invariant gluon source for the operators (1,2) is given by

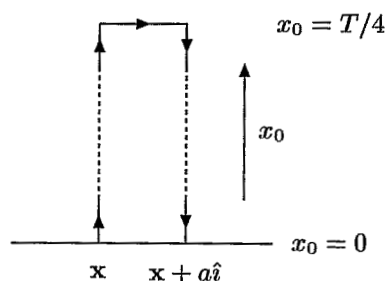
$$\mathcal{S}_g = \text{tr}\{\mathcal{T}_1 \mathcal{T}_2\}$$

where the **big-tooth** state \mathcal{T}_i is defined as

$$\mathcal{T}_i = \frac{a^3}{2i} \sum_{\mathbf{x}} \left\{ \Pi_i(\mathbf{x}) - \Pi_i^\dagger(\mathbf{x}) \right\}$$

$$\Pi_i(\mathbf{x}) = \frac{1}{ag_0} \prod_{x_0=0}^{\frac{T}{4}-a} U_0(x_0, \mathbf{x}) U_i\left(\frac{T}{4}, \mathbf{x}\right) \prod_{x_0=\frac{T}{4}-a}^0 U_0^\dagger(x_0, \mathbf{x} + a\hat{i})$$

- The gauge invariance of \mathcal{S}_g is due to the globality of the gauge group on the time boundaries.



Correlation functions

- In order to measure the renormalization constants of the operators (1,2), we define the following correlation functions

$$\begin{aligned}
 f_{qq}(x_0, p_1) &= -a^6 \sum_{\mathbf{y}, \mathbf{z}} e^{i\mathbf{p} \cdot (\mathbf{y} - \mathbf{z})} \left\langle \frac{1}{4} \bar{\psi}(\mathbf{x}) \gamma_{[1} \overleftrightarrow{D}_{2]} \psi(\mathbf{x}) \bar{\zeta}(\mathbf{y}) \gamma_2 \zeta(\mathbf{z}) \right\rangle \\
 f_{gg}(x_0, p_1) &= -a^6 \sum_{\mathbf{y}, \mathbf{z}} e^{i\mathbf{p} \cdot (\mathbf{y} - \mathbf{z})} \left\langle \sum_{\rho} \text{tr}\{F_{[1\rho}(\mathbf{x}) F_{\rho 2]}(\mathbf{x})\} \bar{\zeta}(\mathbf{y}) \gamma_2 \zeta(\mathbf{z}) \right\rangle \\
 f_{gq}(x_0) &= \left\langle \frac{1}{4} \bar{\psi}(\mathbf{x}) \gamma_{[1} \overleftrightarrow{D}_{2]} \psi(\mathbf{x}) \text{tr}\{\mathcal{T}_1 \mathcal{T}_2\} \right\rangle \\
 f_{gg}(x_0) &= \left\langle \sum_{\rho} \text{tr}\{F_{[1\rho}(\mathbf{x}) F_{\rho 2]}(\mathbf{x})\} \text{tr}\{\mathcal{T}_1 \mathcal{T}_2\} \right\rangle
 \end{aligned}$$

- Moreover, in order to remove the effects introduced by the sources, we define the two auxiliary correlation functions

$$\begin{aligned}
 f_1 &= -\frac{a^{12}}{L^6} \sum_{\mathbf{u}, \mathbf{v}, \mathbf{y}, \mathbf{z}} \langle \bar{\zeta}'(\mathbf{u}) \gamma_5 \zeta'(\mathbf{v}) \bar{\zeta}(\mathbf{y}) \gamma_5 \zeta(\mathbf{z}) \rangle \\
 G_1 &= \frac{1}{L^8} \langle \mathcal{S}_g \mathcal{S}_g' \rangle
 \end{aligned}$$

SF renormalization conditions

- Renormalization conditions are given on the normalized correlation functions:

$$h_{qq}^R|_{\mu=1/L} = h_{qq}^{(0)}, \quad h_{qg}^R|_{\mu=1/L} = 0$$

$$h_{gg}^R|_{\mu=1/L} = h_{gg}^{(0)}, \quad h_{gq}^R|_{\mu=1/L} = 0$$

where

$$h_{\alpha\beta}^R = \sum_{\gamma=q,g} Z_{\alpha\gamma} h_{\beta\gamma}$$

- Expanding such conditions in powers of g_0 , it is possible to express the renormalization constants in terms of the perturbative expansion of the correlations:

$$Z_{qq}^{(1)}\left(p_1, \frac{x_0}{L}, \frac{a}{L}\right) = -\frac{f_{qq}^{(1)}}{f_{qq}^{(0)}} - m_c^{(1)} \frac{1}{f_{qq}^{(0)}} \frac{\partial f_{qq}^{(0)}}{\partial m_0} + \frac{1}{2} \frac{f_1^{(1)}}{f_1^{(0)}}$$

$$Z_{qg}^{(1)}\left(\frac{x_0}{L}, \frac{a}{L}\right) = -\frac{f_{gq}^{(1)}}{f_{gq}^{(0)}}$$

$$Z_{gq}^{(1)}\left(p_1, \frac{x_0}{L}, \frac{a}{L}\right) = -\frac{f_{qg}^{(1)}}{f_{qg}^{(0)}}$$

$$Z_{gg}^{(1)}\left(\frac{x_0}{L}, \frac{a}{L}\right) = -\frac{f_{gg}^{(1)}}{f_{gg}^{(0)}} + \frac{1}{2} \frac{G_1^{(1)}}{G_1^{(0)}}$$

Baryons in Partially Quenched QCD

Martin J. Savage : University of Washington

As lattice calculations are presently performed with quark masses that are significantly larger than those of nature, a rigorous procedure is required to allow for an extrapolation in the quark masses. In QCD, chiral perturbation theory (χ PT) has been studied for the last thirty years or so. It is a double expansion in the external momentum and the light quark masses, m_q , about the $SU(2)_L \otimes SU(2)_R$ chiral symmetry limit of QCD. The pseudo-Goldstone boson sector has been studied in great detail, with order $\mathcal{O}(p^4)$ calculations routine and some have been performed at order $\mathcal{O}(p^6)$. The power counting for this sector is well understood with $m_q \sim p^2$, and the expansion is in powers of p^2 . Heavy-Baryon- χ PT (BH χ PT) [1] has been developed to compute observables involving one baryon. The power-counting is a little more involved with the appearance of v^μ , the four-velocity of the heavy baryon. The expansion for this theory is in powers of p , and converges more slowly than in the meson sector. As the Δ -resonances are only ~ 300 MeV more massive than the nucleons they must be included as a dynamical field in order to describe processes with momentum up to Λ_χ , the chiral symmetry breaking scale. Progress has been made in describing multi-nucleon systems [2], but this is far more complicated than the single nucleon sector.

The idea of partial quenching [3] is that the light quarks (valence quarks) are fully quenched by the addition of bosonic quarks (ghosts). Sea-quarks are added with masses that are greater than or equal to the valence masses. If they are equal to the valence masses then one recovers QCD. The utility of this scenario is that the quenched valence quarks can be kept much lighter than the sea quarks, and thus increase the speed of numerical simulations, which can be extrapolated to nature using partially quenched χ PT (PQ χ PT). The chiral symmetry group for the two-flavor theory ($N_f = 2$) is $SU(4|2)_L \otimes SU(4|2)_R$. Mesonic observables had been studied previously and in works with Jiunn-Wei Chen and Silas Beane [4], I have included the lowest-lying baryon octet and decuplet (or nucleon doublet and Δ -resonances for $N_f = 2$) into PQ χ PT. Using the symmetry properties of an interpolating field for the nucleon we uniquely embedded ghost- and sea- baryons into PQ χ PT so as to implement the addition of both ghost-quarks and sea-quarks. We have computed baryon masses, magnetic moments, the matrix elements of isovector twist-2 operators and axial matrix elements. In extending electromagnetism (or any external probe) from QCD to PQQCD there is a freedom in the charge assignments of the ghosts and sea quarks. It is possible that by a suitable choice of the charges will improve the extrapolation of lattice simulations. Beane and I realized that the nucleon-nucleon potential will be modified at long-distances away from the QCD limit. The Yukawa behavior of QCD is replaced by exponential fall off in PQQCD, and this may have significant impact on efforts to extract properties of multi-nucleon systems from the lattice. Finally, the flavor-conserving parity violation (PV) part of the standard model is poorly known and continues to be a focus of the nuclear physics community. I believe that the only way to make any sort of meaningful comparison between the future experimental determinations of $h_{\pi NN}^{(1)}$ (the momentum independent PV nucleon-pion coupling) and theory is to perform a lattice calculation of the relevant matrix elements.

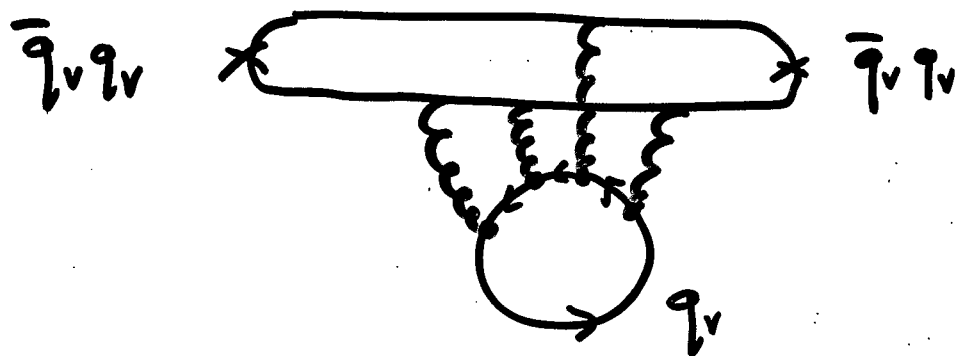
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- [3] S. R. Sharpe and N. Shoresh, *Phys. Rev.* **D62**, 094503 (2000); *Nucl. Phys. Proc. Suppl.* **83**, 968 (2000); M. F. L. Golterman and K.-C. Leung, *Phys. Rev.* **D57**, 5703 (1998). S. R. Sharpe, *Phys. Rev.* **D56**, 7052 (1997); C. W. Bernard and M. F. L. Golterman, *Phys. Rev.* **D49**, 486 (1994).
- [4] J.-W. Chen and M.J. Savage, [hep-lat/0111050](#); S.R. Beane and M.J. Savage, [hep-lat/0203003](#); S.R. Beane and M.J. Savage, [hep-lat/0202013](#).

$$\underline{\underline{N_F = 2}}$$

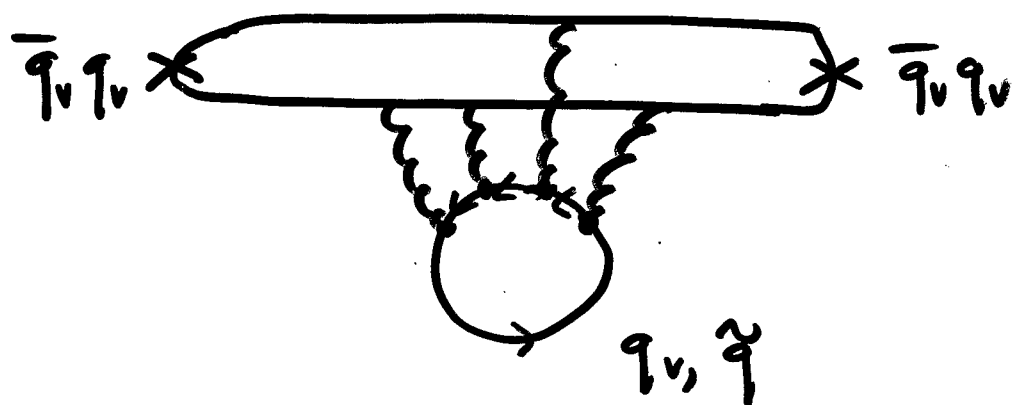
QCD

2 quarks



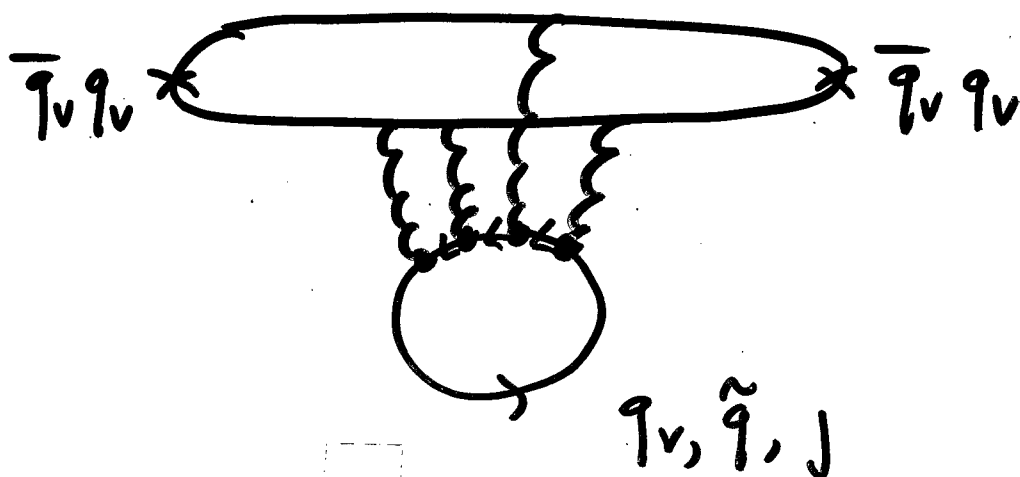
CQCD

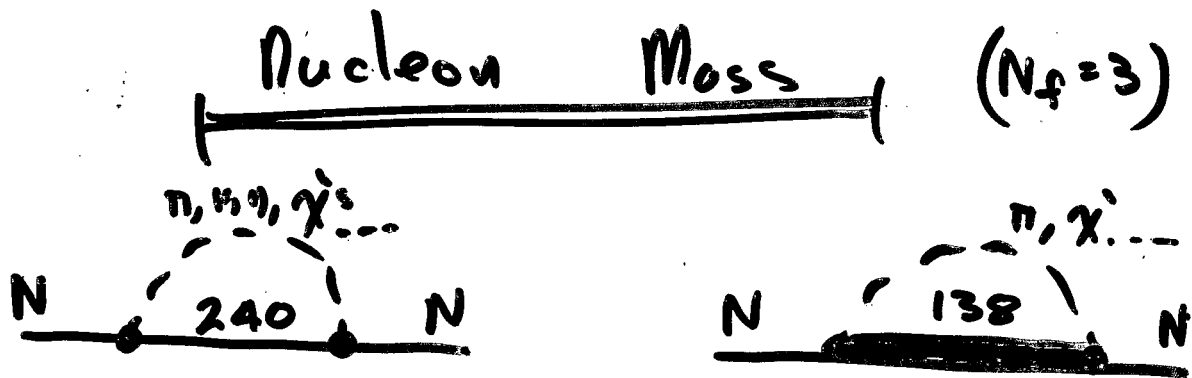
2 quarks + 2 ghost quarks : $m_{q_v} = m_{\tilde{q}}$



PQCD

4 quarks + 2 ghost quarks : $m_{q_v} = m_{\tilde{q}} \leq m_J$





$$\begin{aligned}
 M_N = M_0 &\Rightarrow 2\bar{m}(\alpha_m + \beta_m) \Rightarrow 2\bar{\sigma}_m(2m_j + m_r) \\
 &- \frac{1}{8\pi f^2} \left(4D(F - \frac{D}{2})M_\pi^3 + \frac{1}{3}(5D^2 - 6DF + 9F^2)(2m_{ju}^3 + m_{ru}^3) \right. \\
 &\quad \left. + (D - 3F)^2 G_{\pi,\pi} + \frac{2c^2}{3\pi} (F_{ju} + F_\pi + \frac{1}{2}F_{ru}) \right)
 \end{aligned}$$

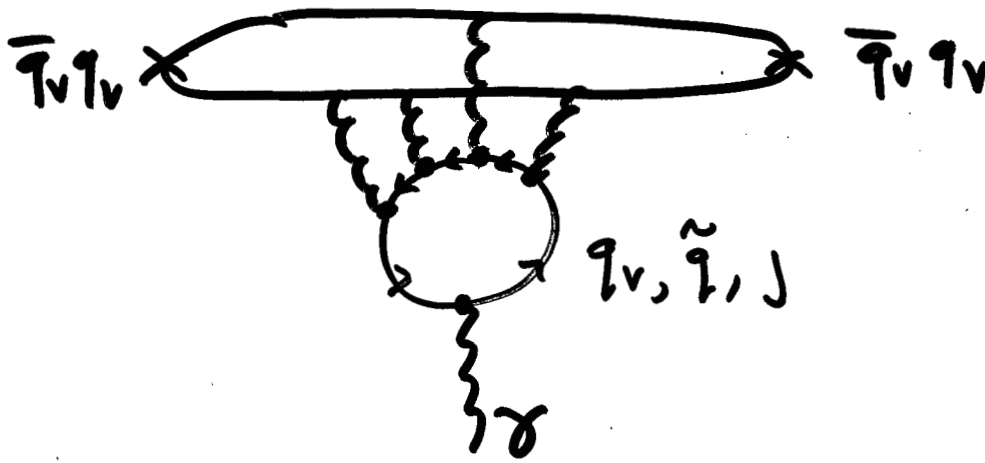
sea-quarks j, l, r

isospin limit $M_u = m_j \neq m_s \neq \bar{m}$
 $m_j = m_l \neq m_r$

As $m_j = m_l \rightarrow \bar{m}$ and $m_r \rightarrow m_s$

\Rightarrow Recover QCD expression.

Charges ?



$$N_f = 3 \quad Q = \frac{1}{3} \begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix} \quad \underline{8} \text{ under } su(3)$$

$${}^{Pa}Q = \text{diag} \left(\frac{2}{3}, \frac{1}{3}, \frac{1}{3}, 1, 1, 1, 1, 1, 1 \right)$$

$$\text{Tr}({}^{Pa}Q) = 0 \quad \Rightarrow \text{adjoint}$$

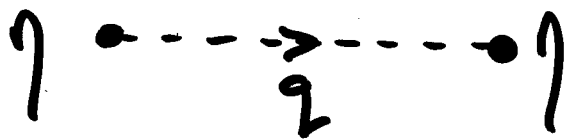
$$m_j \rightarrow m_u, m_i \rightarrow m_d, m_r \rightarrow m_s$$

$$\Rightarrow PQCD \rightarrow QCD$$

$$\Rightarrow \text{EM observables recovered}$$

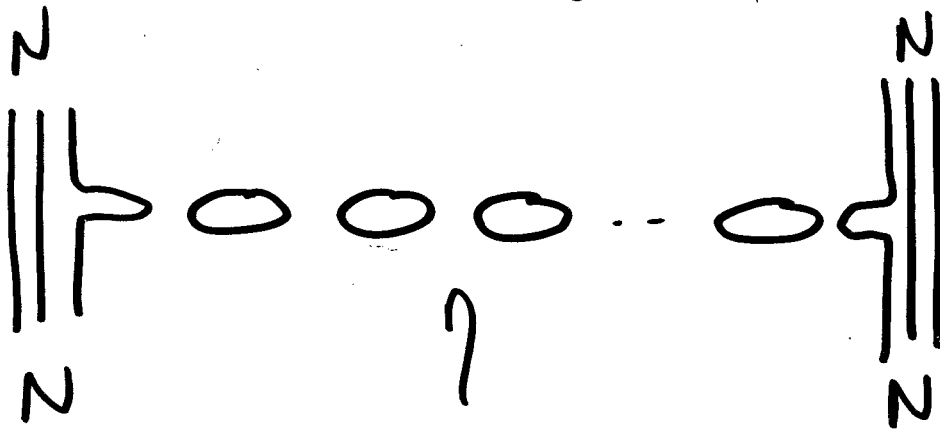
\Rightarrow Different weightings of disconnecteds.

N-N on lattice ? ($N_F=2$) (S. Beane + MJS)



SU(2) singlet

$$G_{\eta\eta}(q^2) = \frac{i(M_{\eta'}^2 - M_\pi^2)}{(q^2 - M_\pi^2 + i\epsilon)^2}$$



$$V^{(PA)}(r) = \frac{1}{8\pi f^2} \sigma_1 \cdot \nabla \sigma_2 \cdot \nabla \left(g_A^2 \frac{\vec{r}_1 \cdot \vec{r}_2}{r} - g_0^2 \frac{M_{\eta'}^2 - M_\pi^2}{2M_\pi} \right) e^{-m_\pi r}$$

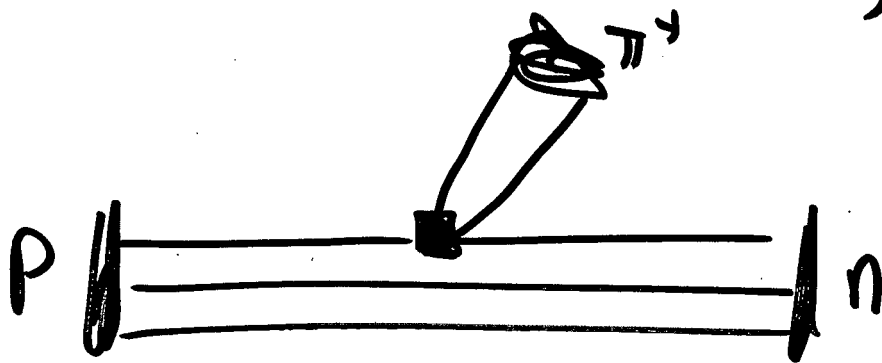
Parity-Violation (S Beane + MJS)

$$\overline{\{Z^0\}}, \quad \overline{\{W\}}$$

$$\mathcal{O} \sim (\bar{u}u - \bar{d}d)_A (\bar{u}u + \bar{d}d)_V$$

$$\sim \bar{S}S_A (\bar{u}u + \bar{d}d)_V$$

+ other colour contractions, $A \leftrightarrow V$ etc.



$$\mathcal{L} \sim h_{\pi NN}^{(1)} \bar{N} [X_L^3 - X_R^3] N$$

$$\sim h_{\pi NN}^{(1)} [\bar{p}n\pi^+ - \bar{n}p\pi^-]$$

Su(2)

$$X_L^3 = \frac{1}{2} \tau^3$$

$$h_{\pi NN}^{(1)} \sim 0 ???$$

What does lattice say

Non-perturbative renormalization in the finite box

Sinya Aoki

Institute of Physics, University of Tsukuba, Tsukuba, Ibaraki
305-8571, Japan

We have formulated the renormalization scheme in the finite box for the domain-wall QCD. We have numerically tested this scheme, showing not only that the scale independent renormalization factors can be evaluated but also that the effect of the explicit chiral symmetry breaking for DWQCD can be easily estimated. We have then applied it to the calculation of the non-perturbative renormalization factors for vector and axial-vector currents in quenched DWQCD with both plaquette and renormalization group improved actions. The dependence of these renormalization factors on the domain-wall height M are investigated in the wide range of bare gauge couplings.

Content

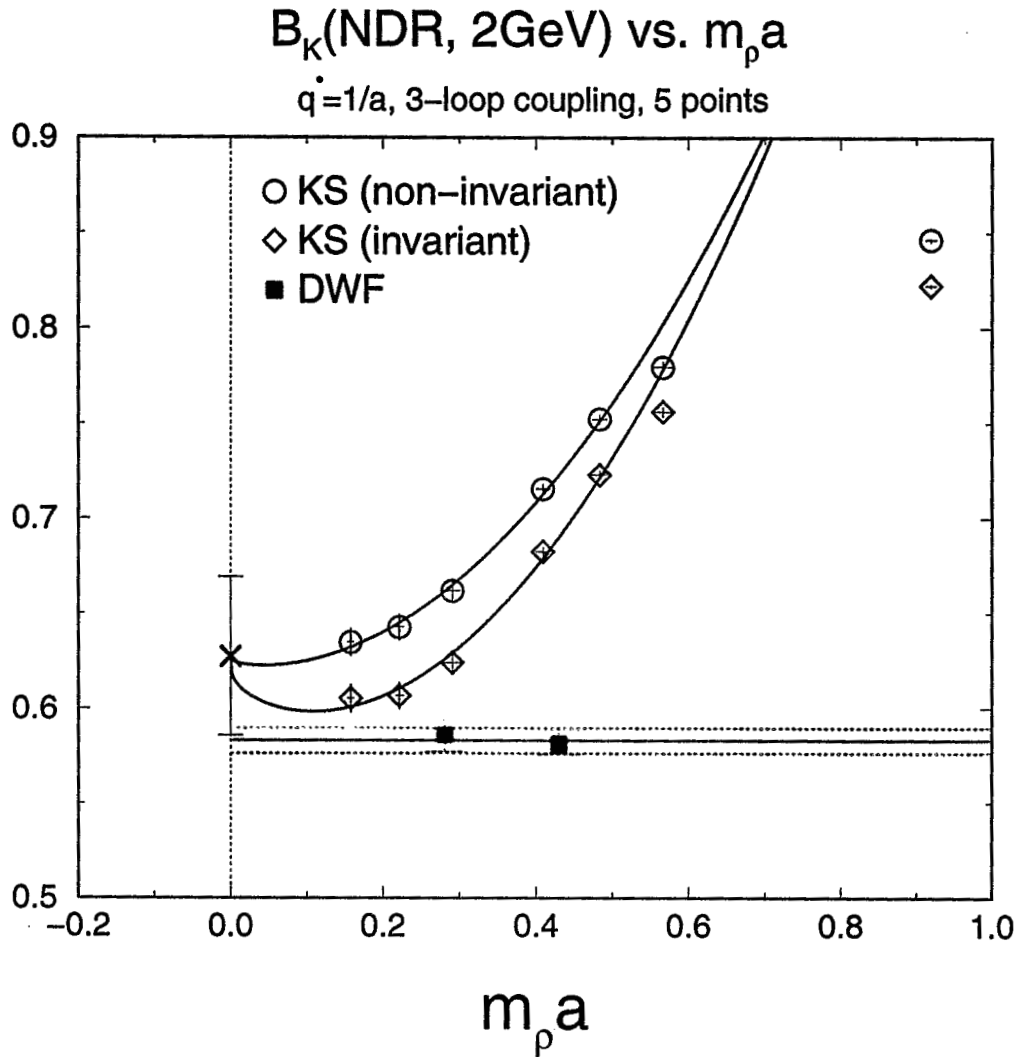
- Domain-wall fermion (DWF)
- Non-perturbative renormalization (NPR)
- DWQCD in the finite box
- Normalization from WT identity
- Test of the method
- Results
- Conclusion

Domain-wall fermion (DWF)

Kaplan, Shamir, Furman-Shamir

good chiral behaviour \longrightarrow good scaling behaviour

B_K in DWQCD



CP-PACS collaboration

$$B_K = \frac{\langle \bar{K}^0 | \bar{s} \gamma_\mu (1 - \gamma_5) d \cdot \bar{s} \gamma_\mu (1 - \gamma_5) d | K^0 \rangle}{\frac{8}{3} \langle \bar{K}^0 | \bar{s} \gamma_\mu \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_\mu \gamma_5 d | K^0 \rangle}$$

Perturbative renormalization

\Rightarrow one of the remaining uncertainties

DWQCD in the finite box

Action on $L^3 \times T \times N_s$

$$S_F = \bar{\psi}(x, s) D(x, s; y, t) \psi(y, t)$$

$X = (x, s), Y = (y, t)$: 5-dim. coordinates

$$D(X, Y) = (5 - M) \delta_{XY} - D^4(x, y) \delta_{st} - D^5(s, t) \delta_{xy}$$

$$D^4(x, y) = P_{-\mu} U(x)_\mu \delta_{y, x+\mu} + P_\mu U_{y, \mu}^\dagger \delta_{y, x-\mu},$$

$$D^5(s, t) = \begin{cases} P_L \delta_{t, s+1} - m_f P_R \delta_{s, N_s} & (s = 1) \\ P_L \delta_{t, s+1} + P_R \delta_{t, s-1} & (1 < s < N_s) \\ -m_f P_L \delta_{t, 1} + P_R \delta_{t, s-1} & (s = N_s) \end{cases}$$

$$P_{\pm\mu} = \frac{1}{2}(1 \pm \gamma_\mu), P_{R/L} = \frac{1}{2}(1 \pm \gamma_5)$$

Quark field

$$q(x) = P_L \psi(x, 1) + P_R \psi(x, N_s) \equiv \sum_s P_L(s) \psi(x, s)$$

$$\bar{q}(x) = \bar{\psi}(x, N_s) P_L + \bar{\psi}(x, 1) P_R \equiv \sum_s \bar{\psi}(x, s) P_R(s)$$

$$P_L(s) = P_L \delta_{s, 1} + P_R \delta_{s, N_s}, P_R(s) = P_R \delta_{s, 1} + P_L \delta_{s, N_s}.$$

Boundary condition

$$\psi(\vec{x}, x_0 = 0, s) = P_+ P_L(s) \rho(\vec{x})$$

$$\bar{\psi}(\vec{x}, x_0 = 0, s) = \bar{\rho}(\vec{x}) P_R(s) P_-$$

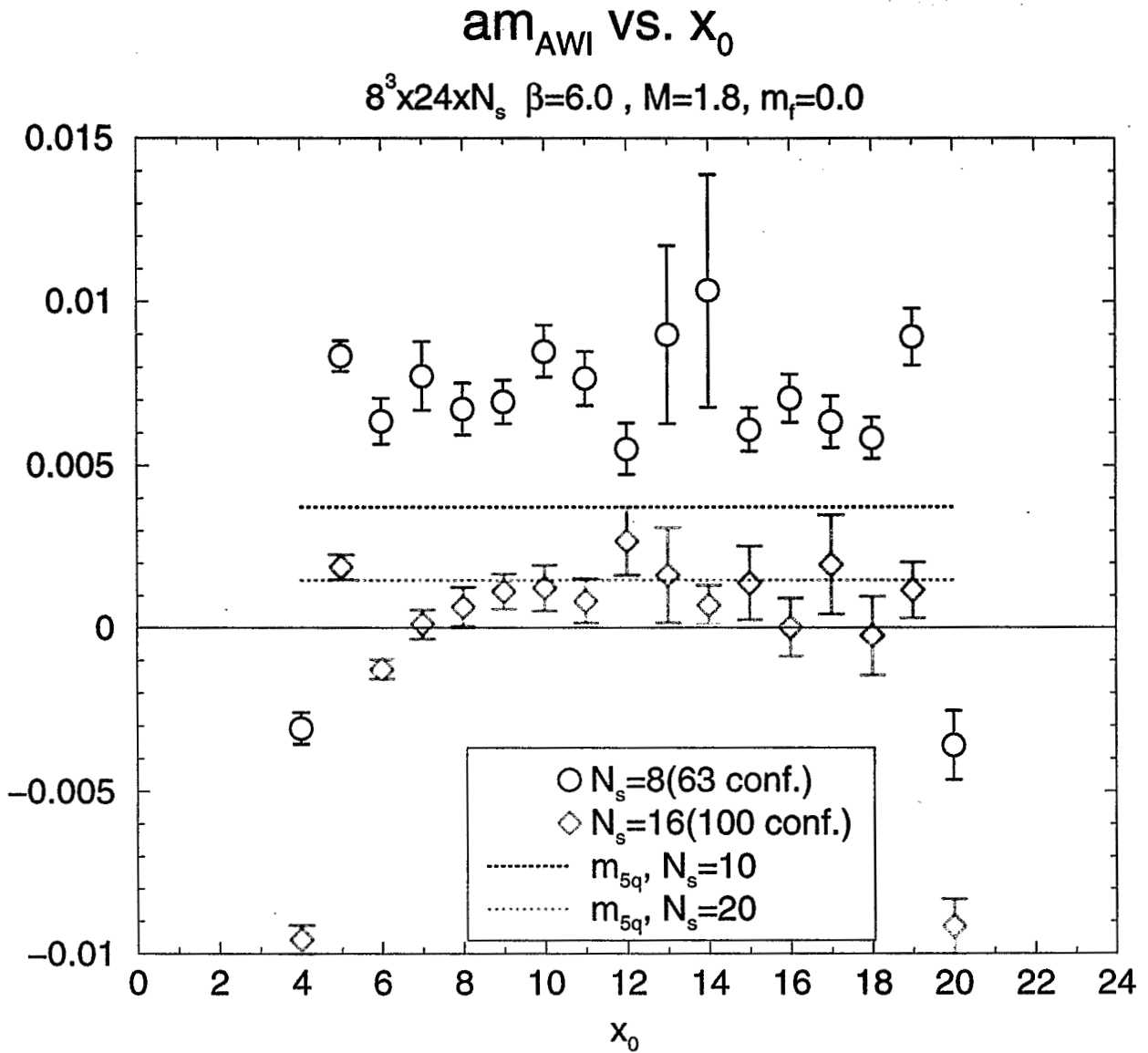
$$\psi(\vec{x}, x_0 = T, s) = P_- P_L(s) \rho'(\vec{x})$$

$$\bar{\psi}(\vec{x}, x_0 = T, s) = \bar{\rho}'(\vec{x}) P_R(s) P_+$$

$$P_\pm = P_{\pm 0}$$

The choice of BC is not unique. This BC may not correspond to the ordinary SF boundary in the continuum limit.

Test of the method (quench at $\beta = 6.0$)



- x_0 dependence is weak away from the boundaries
- Explicit breaking of chiral symmetry becomes smaller for larger N_s
- Size of m_{AWI} is consistent with m_{5q} from the conserved current of DWQCD
- m_{AWI} may be used as the measure of the explicit chiral symmetry breaking (easier than m_{5q})

Global parameterization

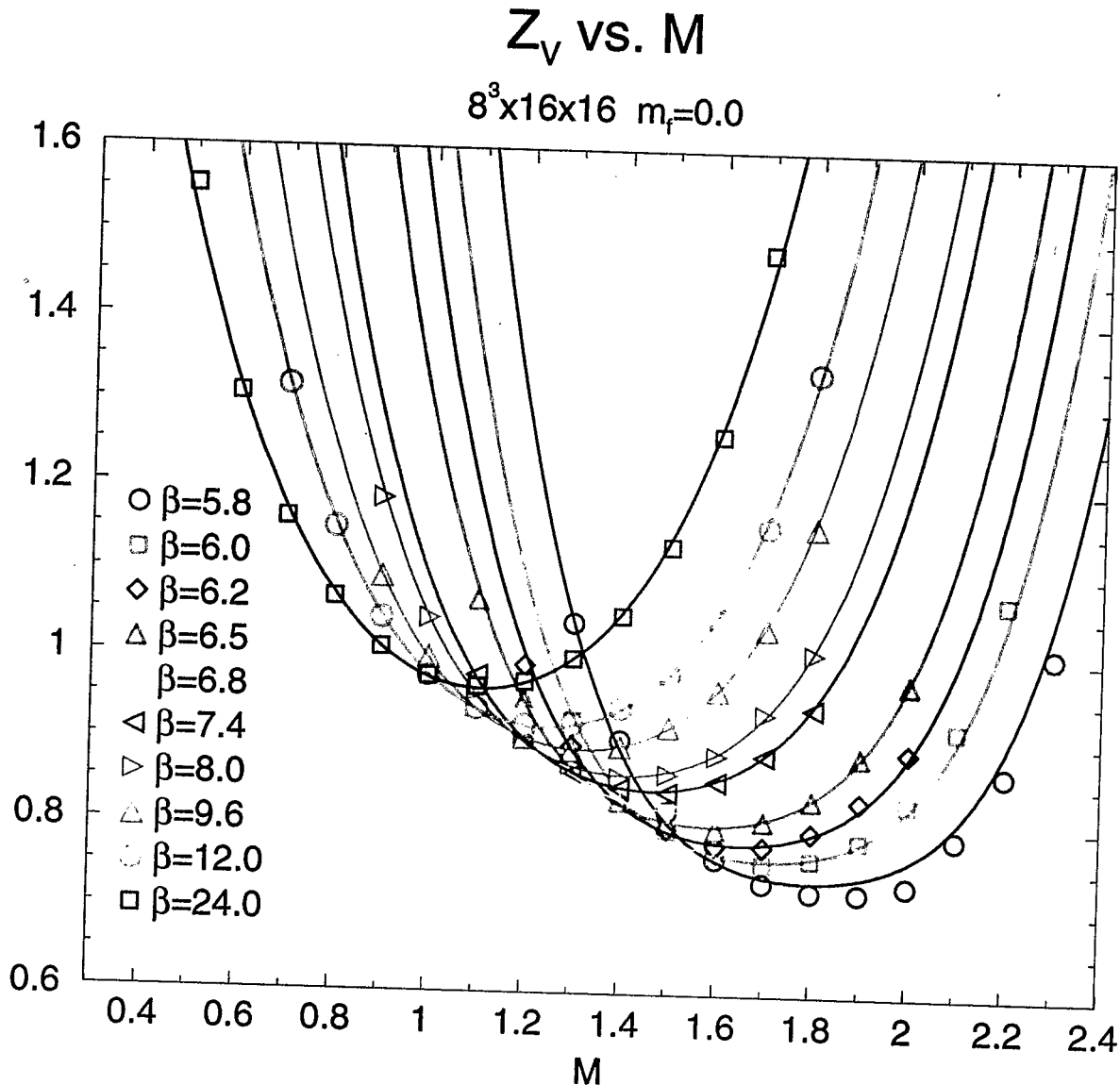
$$Z_V(g^2, M) = A_0 + A_2(M - M_0)^2 + A_4(M - M_0)^4$$

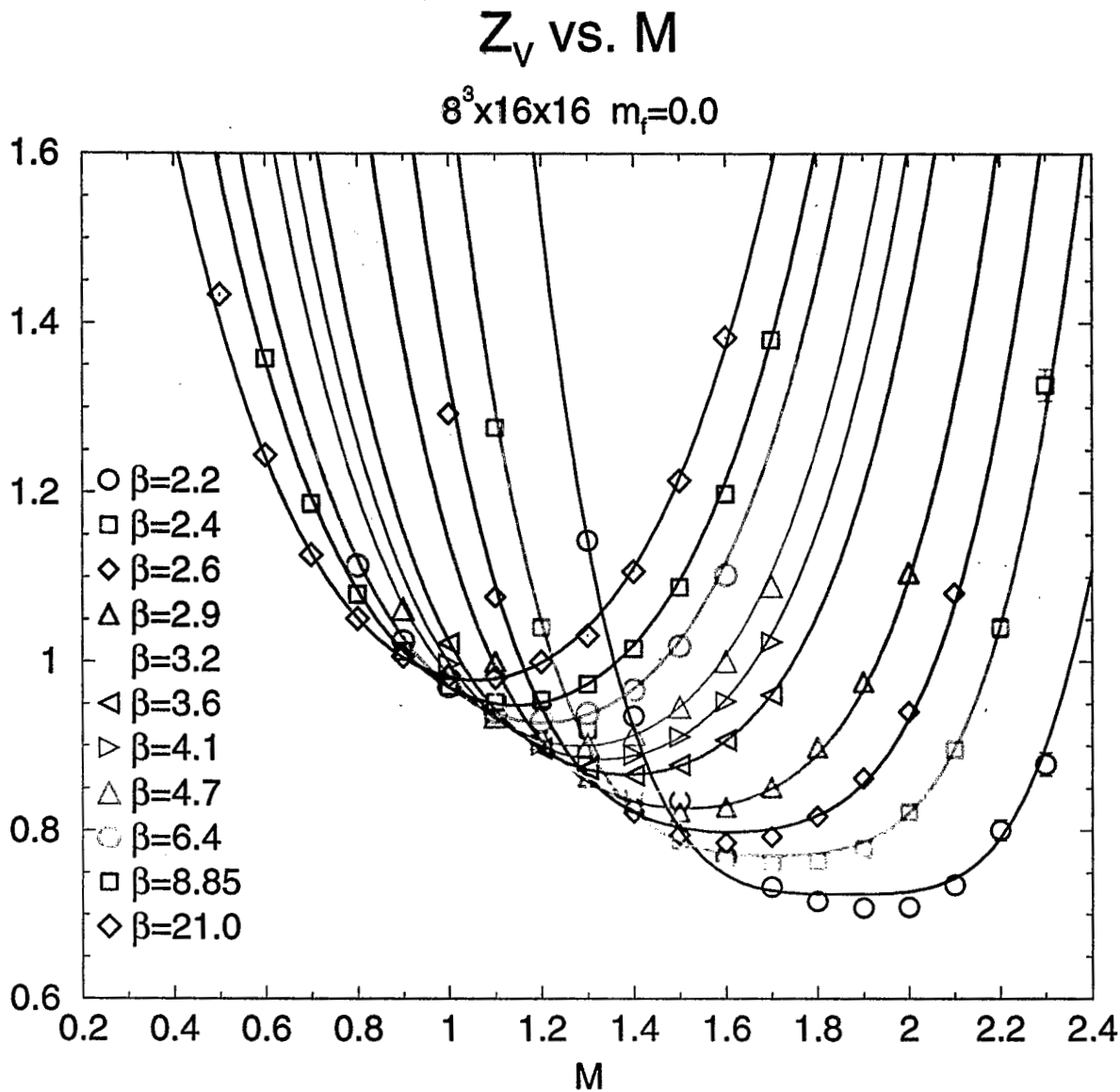
$$A_1 = \frac{1 + b_1 g^2 + c_2 g^4}{1 + a_1 g^2}, \quad A_2 = \frac{1 + b_2 g^2 + c_3 g^4}{1 + a_2 g^2}$$

$$A_4 = \frac{1}{1 + a_2 g^2}, \quad M_0 = \frac{1 + b_0 g^2 + c_0 g^4}{1 + a_0 g^2}$$

with $b_1 - a_1 = A_1(1\text{-loop})$, $b_0 - a_0 = M_0(1\text{-loop})$.

Plaquette action





Conclusions

- $Z_{V,A}$ for DWQCD is non-perturbatively determined in the finite box
- the explicit chiral symmetry breaking of DWQCD can be investigated at $m_f a = 0$ in this method
- the modified boundary condition for DWF should be invented for scale dependent renormalization factors of quark mass and B_K

Breaking the multi-loop barrier in lattice perturbation theory

Paul Rakow

Regensburg

March 2002



Introduction

There are several reasons to feel dissatisfied with conventional lattice perturbation theory. One-loop results are not good enough at the couplings which we use in practice. Their usefulness can be improved somewhat by "tadpole improvement" as suggested by Lepage and Mackenzie (Phys. Rev. D 48, 2250-2264 (1993)), but the perturbative result is still not very predictive unless you have higher loop results.

In some ways we are even making negative progress. A one loop calculation for clover fermions is several times more complicated than a one loop calculation for Wilson fermions. Since the rediscovery of Ginsparg-Wilson fermions, lattice actions are becoming much more complicated, and much less standardised.

One possibility is the "Nonperturbative 'lattice perturbation theory'" of hep-lat/9412100 (Dimm, Lepage and Mackenzie), using Monte-Carlo calculations at very high β to find perturbative coefficients (for the current state of this approach, see Trottier, Shakespeare, Lepage and Mackenzie, hep-lat/0111028).

The approach I want to talk about today is stochastic perturbation theory, using Langevin dynamics to generate perturbation series. This was pioneered by an Italian group, for an early paper see Di Renzo et. al., "Four loop result in SU(3) lattice gauge theory by a stochastic method: lattice correction to the condensate", Nucl.Phys.B426:675-685,1994. In the years since, they have extended this calculation up to 10 loops, "A consistency check for renormalons in lattice gauge theory: β^{-10} contributions to the SU(3) plaquette", hep-lat/0011067.

I want to show that by combining the tadpole improvement / boosted perturbation theory idea of Lepage and Mackenzie with the stochastic perturbation theory techniques of the Italian group one can get results with the precision to answer longstanding questions, and that the combined technique has a promising future.

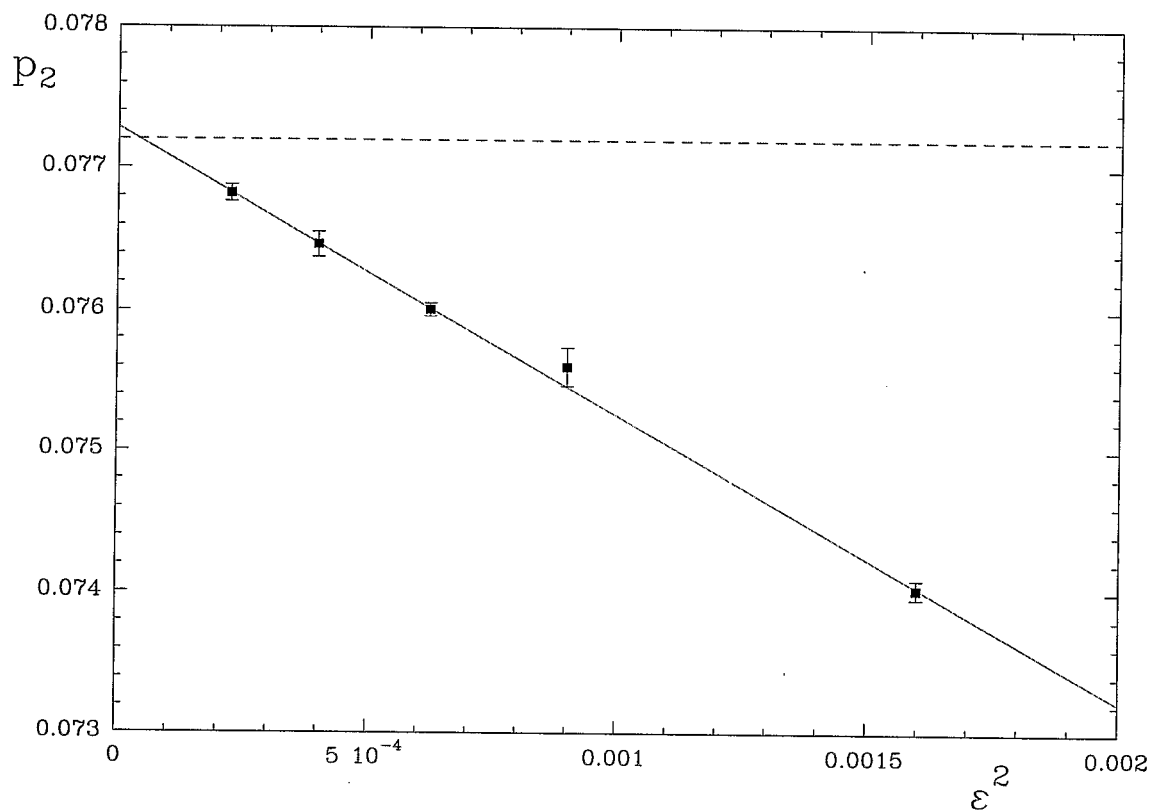
Discretisation of Langevin time

Replace differential equation with a difference equation.

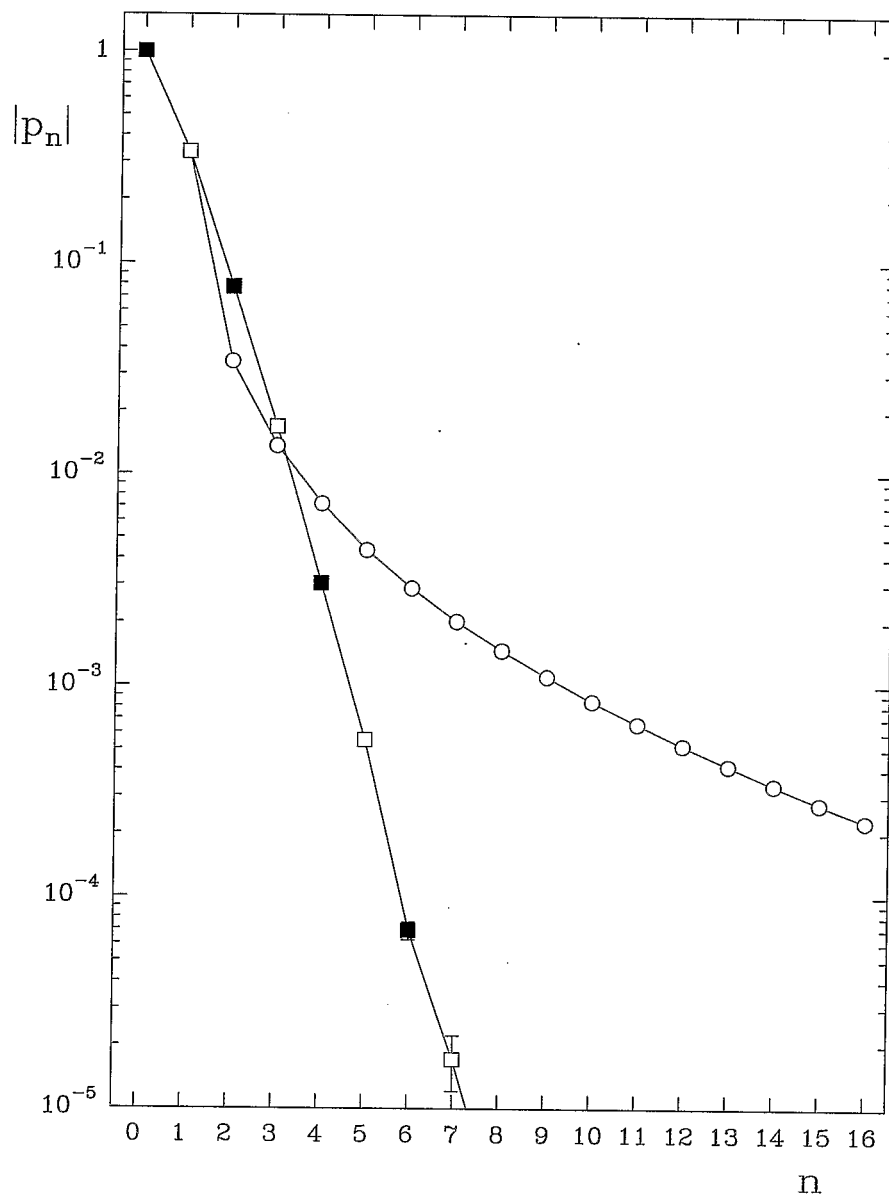
Error proportional to a power of the time-step ϵ .

I use a Runge-Kutta method (similar to Drummond + Horgan). Errors proportional to ϵ^2 .

Several ϵ values, extrapolate to $\epsilon = 0$.



After



Bare coupling series out to 16 loops, tadpole improved series with better accuracy.

HADRON STRUCTURE FROM LATTICE QCD

A RIKEN BNL Research Center Workshop

March 18-22, 2002

LIST OF REGISTERED PARTICIPANTS

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Hadron Structure From Lattice QCD

March 18-22, 2002



RIKEN-BNL Research Center
Brookhaven National Laboratory
Upton, New York

Organizers: Tom Blum, Daniel Boer, Mike Creutz, Shigemi Ohta, Kostas Orginos

| AGENDA | | |
|--------------------------|-------------------|--|
| Monday, March 18 | | |
| 9:00 am - 9:15 am | Mike Creutz | Welcome |
| 9:15 am - 10:15 am | Gerrit Schierholz | Hadron Structure from QCDSF: A Status Report |
| 10:30 am - 10:45 am | Break | |
| 10:45 am - 11:45 am | Stanley Brodsky | Light Front Methods and Non-Perturbative QCD |
| 12:00 am - 12:20 pm | George Sterman | Factorization, local and nonlocal operators |
| 12:35 am - 2:00 pm | Lunch | |
| 2:00 pm - 2:40 pm | Xiangdong Ji | Chiral aspects of parton physics |
| 2:55 pm - 3:25 pm | Akio Ogawa | Transversity measurements at RHIC and KEK |
| 3:40 pm - 4:10 pm | Yuji Goto | Helicity distributions at RHIC |
| Tuesday, March 19 | | |
| 9:00 am - 9:30 am | Jiunn-Wei Chen | Chiral corrections to nucleon parton distributions |
| 9:45 am - 10:25 am | Wally Melnitchouk | Connecting structure functions on the lattice with phenomenology |
| 10:40 am - 11:00 am | Break | |
| 11:00 am - 12:00 am | Keh-Fei Liu | Chiral Symmetry with Overlap Fermions |
| 12:15 pm - 1:30 pm | Lunch | |
| 1:30 pm - 2:00 pm | David Richards | Excited Baryon Spectroscopy |

| | | |
|------------------------|---|---|
| Wednesday, March 20 | | |
| 9:00 am - 10:00 am | John Negele | Calculation of Nucleon Light Cone Quark Distributions in Full Lattice QCD |
| 10:15 am - 10:55 am | Pietro Faccioli | Instanton Contribution to the Hadronic Masses and Electro-Magnetic Form Factors |
| 11:10 am - 11:30 am | Break | |
| 11:30 am - 12:00 pm | Tom Blum | Dirac operator spectrum and topology with domain wall fermions |
| 12:20 pm - 2:00 pm | Lunch | |
| 2:45 pm - 3:15 pm | Kostas Orginos | Improved domain wall fermions |
| 3:15 pm - 3:35 pm | Taku Izubuchi | Lattice QCD with two flavor dynamical domain-wall quarks |
| Thursday, March 21 | | |
| 9:00 am - 9:40 am | Chris Michael | Flavor singlet (and exotic) mesons on the lattice |
| 9:55 am - 10:25 am | Shoichi Sasaki | Isovector axial charge of the nucleon from lattice QCD |
| 10:40 am - 11:00 am | Break | |
| 11:00 am - 11:40 am | Daniel Boer | Lightcone nonlocality and lattice QCD |
| 12:00 am - 1:30 pm | Lunch | |
| 1:30 pm - 2:00 pm | Rich Brower | Hard scattering at wide Angles for QCD strings in AdS space |
| 2:15 pm - 2:45 pm | Sinya Aoki | Heavy quarks (informal talk) |
| 3:00 pm - 3:30 pm | Chris Dawson | Non-perturbative Renormalisation with DWF |
| 7:00 pm - | Workshop dinner – Bellport Country Club | |
| Friday, March 22 | | |
| 9:00 am - 9:30 am | Filippo Palombi | One loop average momentum of singlet parton densities in the Schroedinger Functional Scheme |
| 9:45 am - 10:25 am | Martin Savage | Baryons in partially quenched QCD |
| 10:40 am - 11:00 am | Break | |
| 11:00 am - 11:30 am | Sinya Aoki | Non-perturbative renormalization in the Schrodinger functional scheme |
| 11:45 am - 12:15 am | Paul Rakow | Renormalisation and operator improvement in lattice QCD |
| 12:30 am - 2:00 pm | Lunch | |
| 2:00 pm | Open Discussion and Meeting Adjournment | |

Additional RIKEN BNL Research Center Proceedings:

- Volume 41 – Hadron Structure from Lattice QCD – BNL-
- Volume 40 – Theory Studies for RHIC-Spin – BNL-52662
- Volume 39 – RHIC Spin Collaboration Meeting VII – BNL-52659
- Volume 38 – RBRC Scientific Review Committee Meeting – BNL-52649
- Volume 37 – RHIC Spin Collaboration Meeting VI (Part 2) – BNL-52660
- Volume 36 – RHIC Spin Collaboration Meeting VI – BNL-52642
- Volume 35 – RIKEN Winter School – Quarks, Hadrons and Nuclei – QCD Hard Processes and the Nucleon Spin – BNL-52643
- Volume 34 – High Energy QCD: Beyond the Pomeron – BNL-52641
- Volume 33 – Spin Physics at RHIC in Year-1 and Beyond – BNL-52635
- Volume 32 – RHIC Spin Physics V – BNL-52628
- Volume 31 – RHIC Spin Physics III & IV Polarized Partons at High Q^2 Region – BNL-52617
- Volume 30 – RBRC Scientific Review Committee Meeting – BNL-52603
- Volume 29 – Future Transversity Measurements – BNL-52612
- Volume 28 – Equilibrium & Non-Equilibrium Aspects of Hot, Dense QCD – BNL-52613
- Volume 27 – Predictions and Uncertainties for RHIC Spin Physics & Event Generator for RHIC Spin Physics III – Towards Precision Spin Physics at RHIC – BNL-52596
- Volume 26 – Circum-Pan-Pacific RIKEN Symposium on High Energy Spin Physics – BNL-52588
- Volume 25 – RHIC Spin – BNL-52581
- Volume 24 – Physics Society of Japan Biannual Meeting Symposium on QCD Physics at RIKEN BNL Research Center – BNL-52578
- Volume 23 – Coulomb and Pion-Asymmetry Polarimetry and Hadronic Spin Dependence at RHIC Energies – BNL-52589
- Volume 22 – OSCAR II: Predictions for RHIC – BNL-52591
- Volume 21 – RBRC Scientific Review Committee Meeting – BNL-52568
- Volume 20 – Gauge-Invariant Variables in Gauge Theories – BNL-52590
- Volume 19 – Numerical Algorithms at Non-Zero Chemical Potential – BNL-52573
- Volume 18 – Event Generator for RHIC Spin Physics – BNL-52571
- Volume 17 – Hard Parton Physics in High-Energy Nuclear Collisions – BNL-52574
- Volume 16 – RIKEN Winter School - Structure of Hadrons - Introduction to QCD Hard Processes – BNL-52569
- Volume 15 – QCD Phase Transitions – BNL-52561
- Volume 14 – Quantum Fields In and Out of Equilibrium – BNL-52560
- Volume 13 – Physics of the 1 Teraflop RIKEN-BNL-Columbia QCD Project First Anniversary Celebration – BNL-66299
- Volume 12 – Quarkonium Production in Relativistic Nuclear Collisions – BNL-52559

Additional RIKEN BNL Research Center Proceedings:

- Volume 11 – Event Generator for RHIC Spin Physics – BNL-66116
- Volume 10 – Physics of Polarimetry at RHIC – BNL-65926
- Volume 9 – High Density Matter in AGS, SPS and RHIC Collisions – BNL-65762
- Volume 8 – Fermion Frontiers in Vector Lattice Gauge Theories – BNL-65634
- Volume 7 – RHIC Spin Physics – BNL-65615
- Volume 6 – Quarks and Gluons in the Nucleon – BNL-65234
- Volume 5 – Color Superconductivity, Instantons and Parity (Non?)-Conservation at High Baryon Density – BNL-65105
- Volume 4 – Inauguration Ceremony, September 22 and Non -Equilibrium Many Body Dynamics – BNL-64912
- Volume 3 – Hadron Spin-Flip at RHIC Energies – BNL-64724
- Volume 2 – Perturbative QCD as a Probe of Hadron Structure – BNL-64723
- Volume 1 – Open Standards for Cascade Models for RHIC – BNL-64722

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RIKEN BNL RESEARCH CENTER

Hadron Structure from Lattice QCD

March 18 – 22, 2002



Li Keran

*Nuclei as heavy as bulls
Through collision
Generate new states of matter.*
T.D. Lee

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Speakers:

| | | | | |
|-------------|------------|---------------|----------------|-------------|
| S. Aoki | T. Blum | D. Boer | S. Brodsky | R. Brower |
| J.-W. Chen | M. Creutz | C. Dawson | P. Faccioli | Y. Goto |
| T. Izubuchi | X. Ji | K.-F. Liu | W. Melnitchouk | J. Negele |
| A. Ogawa | K. Orginos | F. Palombi | P. Rakow | D. Richards |
| S. Sasaki | M. Savage | G. Schierholz | G. Sterman | |

Organizers: T. Blum, D. Boer, M. Creutz, S. Ohta, K. Orginos